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## Scattering of P-Wave in Fluid Saturated Medium

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### ABSTRACT

In this study, the scattering of P-waves propagation in saturated medium are exploited. Diffusive profiles of scattering P-waves are based on Biots elastic theory for saturated porous medium, with the medium fall into two distinctive groups: Insoluble as well as soluble medium. Helmholtz equations are solved to give the elastic wave solution. With the existence of fluid in the medium, the deviation in P-waves velocities is expected from these mediums once the saturation degree is asymmetrical. Higher density of fluid supposed to promote diffusive P-wave while low-density fluid induces non-diffusive P-waves. However, the scattering of P-waves has produced contrary outcomes.

**Key words:** Diffusion, attenuation, scattering waves, saturated-medium, Helmholtz equation

### INTRODUCTION

The problem of surface motion at circular alluvial valley subject to SH waves are solved (Chen *et al.*, 2008). Despite being plane P-waves, the surface motion subject to variational P-waves amplitude is purposed in this study.

It is well known that the soil deposits are modeled as porous solid saturated with fluid in natural. Therefore the study of elastic wave propagation in fluid saturated porous media has been of considerable interest. Li *et al.* (2005) solved the problem of scattering P-waves by circular-arc valleys with saturated soil deposits. However, there have been no analytical solution for the scattering and diffraction of waves by irregular surface. Balideh *et al.* (2009) studied the wave propagation in elastic environments analytically and numerically. Recently, the heuristic approach is applied for studying the linear and nonlinear properties of seismic waves (Ling *et al.*, 2011).

In 1978, the scattering and diffraction of the incident SH wave by an underground inclusion exists in an article concerning an underground circular tunnel. Weighted residual method has been used to revisit the problem of scattering and diffraction of SH wave with respect to an underground cavity of arbitrary shape in a two dimensional elastic half plane (Lee and Monogian, 1995). Zainal *et al.* (2010) gave the analytic solution for the scattering of SH waves by means of Helmholtz equations.

When considering the P or SV waves incidence, the problem becomes much more complicated than the SH wave incidence because of the wave mode conversion during the reflection process. Bessel function has being used for studying the scattering and diffraction of plane P-waves in circular arc alluvial valleys (Liang *et al.*, 2001). Scattering and diffraction of SV-waves in fluid saturated medium has been studied analytically for the attenuation of amplitude and saturation degree (Ling *et al.*, 2010a).

**PROBLEM FORMULATION**

Biots dynamic theory of elastic waves in fluid saturated porous media is used for solving the problem. The governing equation read:

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla (\nabla \cdot \mathbf{u}) + \rho \mathbf{F} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \tag{1}$$

Boundary conditions for:

$$\left. \begin{aligned} \sigma_z &\equiv (\lambda + \mu) \frac{\partial u_z}{\partial z} + \lambda \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) = 0 \\ \tau_{xz} &\equiv \mu \left( \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) = 0 \\ \tau_{yz} &\equiv \mu \left( \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right) = 0 \end{aligned} \right\} \tag{2}$$

Initial conditions for:

$$\mathbf{u}(x, y, z) = 0 \tag{3}$$

$$\frac{\partial u_x}{\partial t} = \frac{\partial u_y}{\partial t} = \frac{\partial u_z}{\partial t} = 0 \tag{4}$$

In linear elasticity for isotropic medium,  $\lambda$  and  $\mu$  denote the Lamé parameters for the stress  $\sigma_z$ ,  $\tau_{xz}$ ,  $\tau_{yz}$  and the displacements  $u_x$ ,  $u_y$  and are continuous everywhere.  $\mathbf{F}$  is the body force in the direction of  $x$ ,  $y$ ,  $z$ , respectively and  $\rho$  is the density.

Provided that  $\omega$  is the angular velocity or frequency,  $k$  is the wavenumber,  $c$  is the wave velocity along with the dispersion relation  $\omega = ck$  the group velocity of a wave is the velocity with the overall shape of the wave's amplitudes which is also known as envelope of the waves. In other words, the phase velocity is the average velocity of the components, given by  $V_p = \omega/k$ . The group velocity is velocity of the envelope, given by  $V_g = d\omega/dk$ .

When this is applied to the problem of P-waves propagation in fluid saturated medium, the group velocity reflects the apparent velocity of the surface displacement or the overall shape of the P-waves amplitude at the boundary of the medium. The envelope is formed by the phase velocity of P-waves. In this research, the apparent velocity  $V_{app}$  or surface displacement velocity is measured along the boundary of the similar density medium in accordance to Snell law:

$$\frac{V_{app}}{\sin f} = \frac{c}{\sin e} \tag{5}$$

where  $e$  is the incident angle and  $f$  is the refraction angle made by the P- and S-waves. However, our objective is to study P-waves in  $x$ -direction only. For the case of fluid-saturated medium, there exists variation in density within the medium (Sharma, 2007). There exists slowness induced by

refraction for fluid-saturated medium i.e., the apparent velocity of the displacement at the boundary is slower than phase velocity of the wave in the medium that reads:

$$V_{app} < c \tag{6}$$

In this study, the elastic wave equation will be solved so as to obtain the P-waves displacements in the different types of medium that satisfied (5) and (6). By using the divergence operator:

$$\nabla^2 \mathbf{u} = \nabla(\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u}) \tag{7}$$

Equation 1 is reduced to:

$$\alpha^2 \nabla(\nabla \cdot \mathbf{u}) - \beta^2 \nabla \times \nabla \times \mathbf{u} + \mathbf{F} = \frac{\partial^2 \mathbf{u}}{\partial t^2} \tag{8}$$

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \beta = \sqrt{\frac{\mu}{\rho}} \tag{9}$$

$\alpha$  and  $\beta$  are the velocities for the P-wave and S-wave and Eq. 8 is the elastic wave equation. Hence, this modeling only valid for elastic medium and it is necessary to reduce the Right Hand Side (RHS) of (8) by letting:

$$\mathbf{u} = \exp[i(\mathbf{kx} - \omega t)] \tag{10}$$

By inserting second order derivative of (10) into RHS of (8), the equation reads:

$$\alpha^2 \nabla(\nabla \cdot \mathbf{u}) - \beta^2 \nabla \times \nabla \times \mathbf{u} + \mathbf{F} = -\omega^2 \mathbf{u} \tag{11}$$

Since elastic wave consists of irrotational by P-wave and solenoid by S-wave, the displacement shall be written as:

$$\mathbf{u} = \mathbf{u}_p + \mathbf{u}_s \tag{12}$$

For irrotational P-waves, the vorticity. Thus, the relation (12) reads:

$$\nabla \cdot \mathbf{u}_p \neq 0, \nabla \times \mathbf{u}_p = 0, \nabla \cdot \mathbf{u}_s = 0, \nabla \times \mathbf{u}_s \neq 0 \tag{13}$$

By inserting Eq. 12 and 13 into 11, the Eq yields:

$$\alpha^2 (\nabla^2 \mathbf{u}_p + k_p^2 \mathbf{u}_p) + \beta^2 (\nabla^2 \mathbf{u}_s + k_s^2 \mathbf{u}_s) = 0 \tag{14}$$

$$\omega_p = \frac{\alpha}{k_p}, \omega_s = \frac{\beta}{k_s}, F = 0 \tag{15}$$

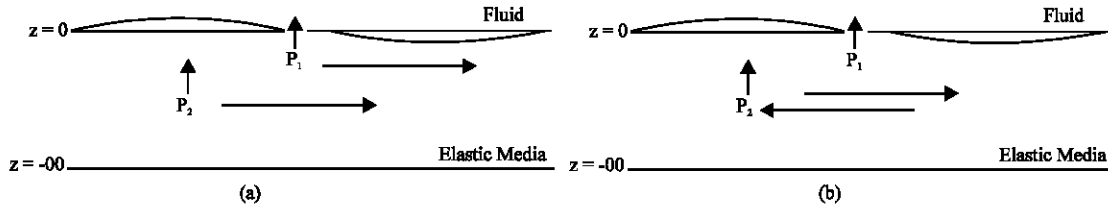


Fig. 1: The propagation of P-waves. (a) The P-waves propagation in similar direction. (b) The P-waves propagate in opposite direction

In the following, the Eq. 16 are the Helmholtz equations for P and S-waves:

$$\nabla^2 u_p + k_p^2 u_p = 0, \nabla^2 u_s + k_s^2 u_s = 0 \tag{16}$$

Here, the Helmholtz equations are solved by utilizing Hansen vector that gives:

$$u_p = A(a_x + \eta_\alpha a_z) \exp[i\omega(t - x + \eta_\alpha z)], \eta_\alpha = \sqrt{\frac{c^2}{\alpha^2} - 1} \tag{17}$$

$a_x$  and  $a_z$  are the unit vectors while  $l$  and  $n$  are the vector components. P-wave vanishes when the depth go infinity as illustrated in Fig. 1a-b.  $\eta_\alpha$  is always positive. The velocity of the P-wave,  $a$  is measured at the boundary of the medium or  $z = 0$  and it is similar to the apparent velocity  $V_{app}$  of the surface displacement.  $c$  is the P-wave velocity in the medium. Here,  $\eta_\alpha$  quantity refers to the refracted P-wave velocity that gives the refracted frequency  $\omega = \eta_\alpha k$  and the vectors  $a_x$  and  $a_z$  illustrates the displacement is polarized between x-z directions. The polarization of Eq.17 gives the frequency Eq that reads:

$$u_p = A\eta_\alpha \exp(ik\eta_\alpha z) \exp[ik(ct - x)], \tan \theta = \frac{\eta_\alpha}{1} \tag{18}$$

Consequently, the P-waves propagate in the positive x-direction. From wave terminology, the term  $\exp(ik\eta_\alpha z) \exp[ik(ct - x)]$  in Eq. 18 will illustartes that the harmonic wave is diffusive if the frequency  $\omega = \eta_\alpha k$  is complex. The diffusive waves are associated with attenuation of the amplitudes with the time due to certain dissipation mechanisms present in the system.

Next, the roles of the refracted velocity  $\eta_\alpha$  will be shown. For the fluid-saturated medium, there comes the slowness induced by refraction (Keith and Crampin, 1977; Wang *et al.*, 2009). The envelope velocity at medium surface is different with wave velocity in the medium after the slowness or  $V_{app} \neq c$ . For a particular case, the P-wave velocity in the medium is greater than the envelope's velocity that yields:

$$c > \alpha, \alpha = V_{app} \tag{19}$$

Here, we propose the relation for another two types of medium conditions such that:

$$c = \alpha, \alpha = V_{app} \tag{20}$$

$$c < \alpha, \alpha = V_{app} \tag{21}$$

Here, we mark that the relations (19) and (21) are meant for the insoluble medium such that the variation of velocities  $c$  and  $a$  is significant. When the velocity  $c$  is similar to  $a$ , we presume this will explain the soluble medium such that the fluid mixes well with the medium to give similar velocity.

Yang and Sato (2000); Kahraman (2007) illustrates that the high-density medium promotes high waves velocity. Hence, the relation (19) will only be present when the low-density fluid is saturated in the insoluble medium; the low-density fluid will reshuffle the ray velocity or reduce the P-wave velocity. Eventually, the relation (21) is meant for the high density fluid saturated in the insoluble medium.

The detail explanations about the tie between mediums solubility and the fluids density will be discussed next for showing the vital roles play by the relations 19-21 especially the displacements characteristic. When condition (21) is applied to the quantity  $\eta_a$  in Eq. 18, a complex solution will be obtained for that reads:

$$u_p = Ai\eta_\alpha \exp(-k\eta_\alpha z)\exp[ik(ct-x)] \text{ with } \eta_p \rightarrow i\eta_p \tag{22}$$

Hence, the P-waves displacements for three types of mediums read:

$$u_p = Ai\eta_\alpha \exp(-k\eta_\alpha z) \exp [ik(ct-x)] \text{ for } c < a \tag{23}$$

$$u_p = A\eta_\alpha \exp [ik(ct-x)] \text{ for } c = a \tag{24}$$

$$u_p = A\eta_\alpha \exp(ik\eta_\alpha z) \exp [ik(ct-x)] \text{ for } c > a \tag{25}$$

Equation 23-25 illustrates that the P-waves propagation in x-direction while the diffusion is in z-direction. The Eq. 23-25 are arranged such that the density of the saturated fluid reduces from  $c < a$  to  $c > a$ . Implementing  $c = a$  to the Eq. 18, the P-wave illustrates non diffusive characteristics for  $\eta_\alpha = 0$ . This wave is similar to the plane wave or acoustic sound wave.

According to the objective of this study, the Eq. 23 and 25 with similar amplitude will be amended to depict the circumstances in Fig. 1a and b. Two different P-waves with different diffusive characteristic but similar propagating direction that reads:

$$u_p = Ai \left\{ \eta_{\alpha 1} \exp(-k\eta_{\alpha 1} z) \exp [ik(c_1 t - x)] + \eta_{\alpha 2} \exp(-k\eta_{\alpha 2} z) \exp [ik(c_2 t - x)] \right\}, c_{1,2} < \alpha_{1,2} \tag{26}$$

$$u_p = A \left\{ \eta_{\alpha 1} \exp(ik\eta_{\alpha 1} z) \exp [ik(c_1 t - x)] + \eta_{\alpha 2} \exp(ik\eta_{\alpha 2} z) \exp [ik(c_2 t - x)] \right\}, c_{1,2} > \alpha_{1,2} \tag{27}$$

Table 1: The selected value for  $\alpha_1, \alpha_2, \alpha_3, c_1$  and  $c_2$

	$c_{1,2} < \alpha_{1,2}$					$c_{1,2} > \alpha_{1,2}$			
	$\alpha_1$	$\alpha_2$	$c_1$	$c_2$		$\alpha_1$	$\alpha_2$	$c_1$	$c_2$
a	4	4	2	2	d	2	2	4	4
b	4	4	1	2	e	1	2	4	4
c	4	2	2	2	f	2	2	2	4

with:

$$\eta_{\alpha 1} = \sqrt{\frac{c_1^2}{\alpha_1^2} - 1}, \eta_{\alpha 2} = \sqrt{\frac{c_2^2}{\alpha_2^2} - 1}$$

For the cases of opposite direction of propagation, the equations read:

$$u_p = A i \left\{ \eta_{\alpha 1} \exp(-k \eta_{\alpha 1} z) \exp[ik(c_1 t - x)] + \eta_{\alpha 2} \exp(-k \eta_{\alpha 2} z) \exp[-ik(c_2 t - x)] \right\} c_{1,2} < \alpha_{1,2} \tag{28}$$

$$u_p = A \left\{ \eta_{\alpha 1} \exp(i k \eta_{\alpha 1} z) \exp[ik(c_1 t - x)] + \eta_{\alpha 2} \exp(i k \eta_{\alpha 2} z) \exp[-ik(c_2 t - x)] \right\} c_{1,2} > \alpha_{1,2} \tag{29}$$

with:

$$\eta_{\alpha 1} = \sqrt{\frac{c_1^2}{\alpha_1^2} - 1}, \eta_{\alpha 2} = \sqrt{\frac{c_2^2}{\alpha_2^2} - 1}$$

## DISCUSSION

Equation 23-25 are derived for the medium with symmetrical density such that there is no variation in waves velocities in both the fluid and the medium. Varying the saturation degree in the medium are asymmetrical, it is expected to observe the variation in P-waves velocities. Hence, the Eq. 26-29 are derived for studying the diffusive profile relating to the P-waves in asymmetrical porosity medium saturated with low and high density.

Conferring to the P-waves propagated in similar direction with the velocities vary due to the fluid density, Fig. 2a-c are plotted for the medium saturated with low density fluid while Fig. 2a-c are generated for showing the medium saturated with higher density of fluid. With the selected parameter for the apparent velocities and the phase velocities from Table 1, these P-waves are found to be diffusive when the apparent velocity  $a_{1,2}$  is greater than the phase velocity  $c_{1,2}$ . In contrast, these P-waves are non-diffusive when the apparent velocity,  $a_{1,2}$  is weaker than the phase velocity  $c_{1,2}$ . (Ling *et al.*, 2010b).

For Fig. 2d, P-waves are non-diffusive when  $\alpha_1 = \alpha_2$  and  $c_1 = c_2$  with  $\alpha_{1,2} < c_{1,2}$ . This indicates that there is possibility of P-waves being plane wave that is similar to sinusoidal wave or sound wave appears in insoluble medium when the velocities for fulfilling  $\eta_\alpha = 0$  is met. When  $\eta_\alpha = 0$ , the scattering of P-waves do not exist seeing as the reflection does not exist in linear medium. Nikolaev (1989) found linear P-waves in linear medium and the linear P-waves are suggested

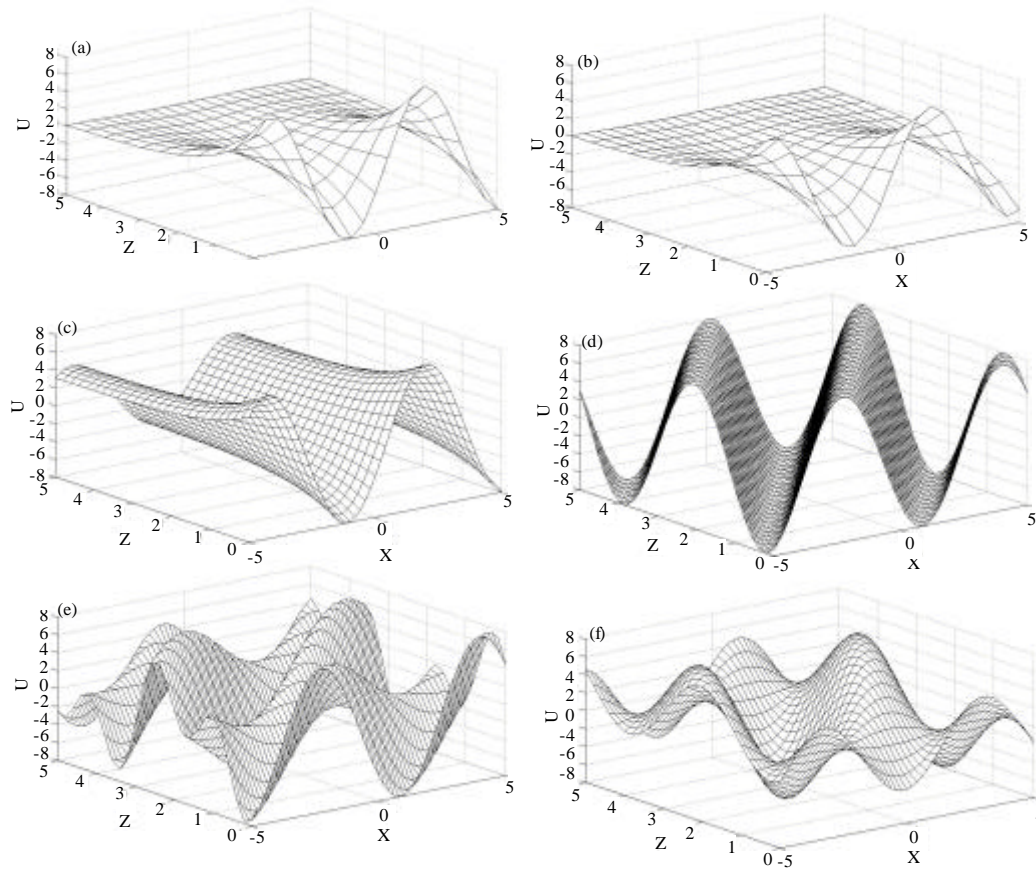


Fig. 2: The diffusive profiles generated by propagating P-waves in similar direction. (a-c) are for the saturated medium with  $c_{1,2} < a_{1,2}$  while (d-f) are for the saturated medium with  $c_{1,2} > a_{1,2}$

similar to sinusoidal waves. Similar waves are also shown in Eq. 24 for soluble medium. Hence, it is well said the soluble medium is linear medium, giving the linear P-waves that is similar to sinusoidal waves.

For the case of P-waves scatter in medium saturated with low density fluid, Fig. 2e-f are generated with the varying  $c_{1,2} > a_{1,2}$  velocities. Despite being non diffusive, P-waves are recursive seeing as the waves diffracted within the medium. Such outcome is difference with the conclusion remarked by Ling *et al.* (2010b) for the situation of non scattering P-waves in saturated medium. Hasheminejad and Avazmohammadi (2007) illustrates the possible of wave diffraction in fluid saturated porous medium. Varying velocities are shown in Table 1 and the non diffusive waves have become recursive once there are varying velocities for. Zhou *et al.* (2008) presented analytic solution for the distribution of dynamic stress that are found for scattering plane wave by the outer boundary of circular-arc valley in a poroelastic half space. Comparing with the result by Zhou *et al.* (2008), distribution of dynamic stress concentration around the outer boundary of circular arc valley is related to these recursive P-waves given that the distribution of stress by the waves are non linear throughout the medium with varying velocities. As explained previously, it is not viable for the sinusoidal and diffusive waves to initiate the nonlinear distribution of dynamic stress revealed by Zhou *et al.* (2008) since the diffraction does not exist.

Figure 3a-f are generated in according to the parameter in Table 1 for the propagating P-waves in opposite direction as illustrated in Fig. 1b. According to the generated Fig. 3a-c, the diffusive



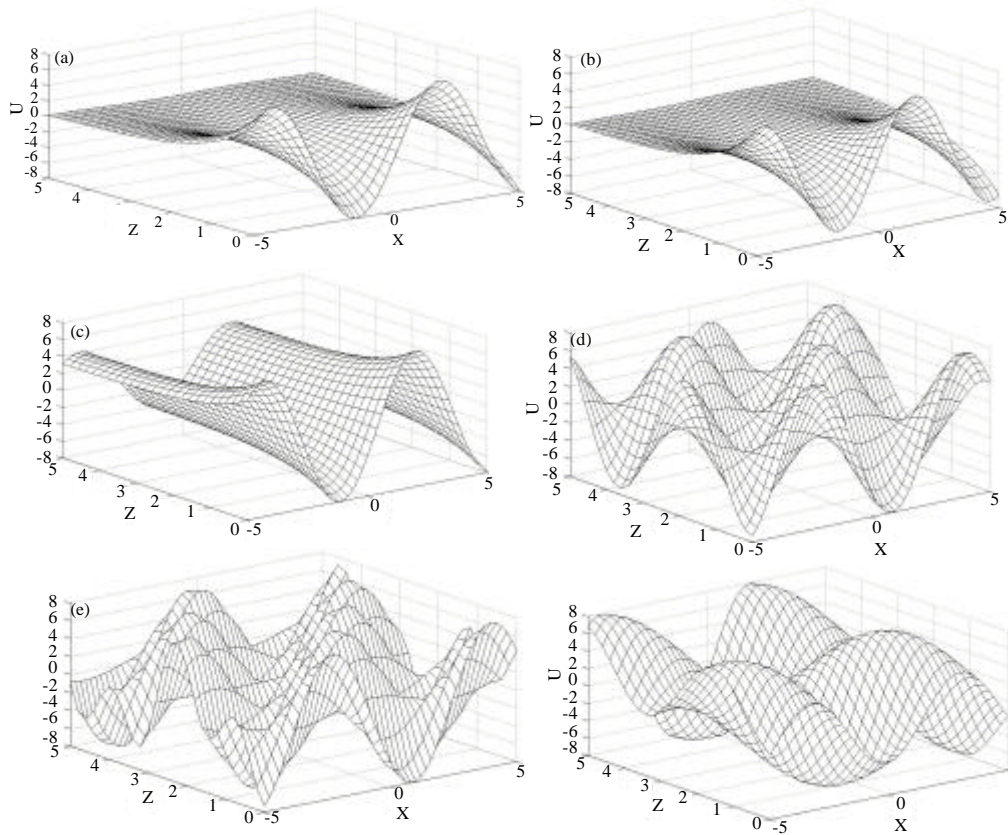


Fig. 3: The diffusive profiles generated by propagating P-waves in opposite direction. (a-c) are for the saturated medium with  $c_{1,2} < a_{1,2}$  while (d-f) are for the saturated medium with  $c_{1,2} > a_{1,2}$

characteristic of P-waves are depicted while the non diffusive P-waves are illustrated through Fig. 3d-f.

However, Fig. 3d does not illustrate the plane wave or sinusoidal wave which is explained in Fig. 2d. With  $\eta_\alpha = 0$  the opposite direction of P-wave does not depict the diffusive characteristic as well. With the medium saturated with low density fluid, it is said the possibility of P-wave for being plane wave will not happen when the P-waves are propagating in opposite direction. Hence, Nikolaev (1989) judgment is only valid for scattering of P-waves in similar directions as illustrated in Fig. 1a.

Apparently, Fig. 3-f are illustrating the scattering of non diffusive P-waves with the amplitude greater than diffusive P-waves. The deviation of amplitude is significant and the waves are recursive. Figure 3d and e are illustrating nearly similar wavelength while Fig. 3f is illustrating a longer wavelength.

## CONCLUSION

It is concluded that the higher density of fluid in the medium will generate diffusive P-waves is firm. Non diffusive P-waves are induced by the medium saturated with lower density fluid, whereas the similar direction of propagating P-wave has tendency to generate sinusoidal wave that is similar to the P-waves in the soluble medium. From the results, it is well said that the opposite

direction of propagating P-wave will not generate sinusoidal wave. Consequently, the instability of medium saturated with lower density of fluid is purposed for generating the contrary outcome. For further research, the outcome is suggested for studying hydrodynamics and soil dynamics.

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