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Meeting the Desired Project Completion Time by Stretching Critical Activities

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ABSTRACT
There are many situations in CPM in which the owners or project managers are crashing the time of project completion to obtain the shortest possible duration to complete a project at least cost within the maximum available budget. In certain circumstances they are forced to extend the completion of the project or reduce the total cost. Generally, the goal of this study is to propose a framework and algorithm for a new strategy that uses trade-off of time against cost to complete the project in the shortest possible duration at least cost within the maximum available budget, then trade-off of cost against time to meet the desired project completion time at least total cost. This is achieved by applying the approach of Stretching Noncritical and Critical Activities (SNCA).

Key words: CPM, time-cost trade-off, crashing conditions, stretching conditions, least cost-scheduling

INTRODUCTION
The objective of CPM is to establish a feasible and desirable relationship between the time and cost of the project by reducing the target time and taking into account the cost of expediting (Nicholas, 2004; O'Brien and Plotnick, 2006; Taha, 2007). Trade-off between project duration and total cost are extensively discussed in the project scheduling literature because of its practical relevance. It is generally, realized that when project duration is compressed, the project will call for an increase in labor and more productive equipment and require more demanding procurement and construction management and then the cost will increase (Abbasnia *et al.*, 2008). Many studies conducted on CPM for time-cost trade-off. Schumacher (1995) discussed the different evaluation techniques using CPM. The article briefly describes the different delay types, assigning responsibility of delays and the special case of concurrent delays. McCullough (1999) discussed delay analysis. Contemporaneous analysis of delays is effective because the best time to evaluate a delay is at the time it occurs. Liberatore (2001) showed that of the construction respondents, 89% used CPM for planning and 72% for control during construction. Anagnostopoulos (2002) explained the effective procedure for time-cost trade off in CPM networks when discrete time-cost combinations are allowed on the project activities is developed. Laslo (2003) showed in his study that a stochastic formulation of time-cost tradeoffs cannot be uniform for all conditions, therefore they must distinguish between different types of activities and different types of cost terms. Bagherpour *et al.* (2006) supposed many traditional cost-time trades off models are computationally

expensive to use due to the complexity of algorithms especially for large scale problems. They present a new approach to adapt linear programming to solve cost time trade off problems. Ipsilandis (2006) supposed the CPM and the Repetitive Scheduling Method (RSM) are the most often used tools for the planning, scheduling and control Linear Repetitive Projects (LRPs). Yang (2007) illustrates in his article the project crashing analysis is to minimize the required cost while meeting a specified deadline. The results illustrate the promising performance of the proposed algorithm. Koo *et al.* (2007) supposed that construction planners face many scheduling challenges during the course of a project. Vanhoucke and Debels (2007) elaborated on three extensions of the well-known discrete time/cost trade-off problem in order to cope with more realistic settings: time/switch constraints, work continuity constraints and net present value maximization. Ke *et al.* (2009) claimed in their study that in real-life projects, both the trade-off between the project cost and the project completion time and the uncertainty of the environment are considerable aspects for decision-makers. The goal of the study is to reveal how to obtain the optimal balanced of the project completion time and the project cost in stochastic environments. Li and Wang (2009) discuss the risk management project is an important aspect of general project risk element transmission theory.

This study provides a framework and algorithm to meet F (the desired project completion time at least total cost) by approach of Stretching Noncritical and Critical Activities (SNCA).

MATERIALS AND METHODS

We begin by crashing all activities in the project simultaneously then stretching the noncritical activities to their normal time until all the slack in the different noncritical paths network is used up to obtain the greatest cost saving. These steps are for completing a project in T (the shortest possible duration to complete the project at least cost within the maximum available budget). Therefore, to meet F, we stretch critical activities to their normal time for increasing the project completion time. Stretching the critical activity that has the biggest cost slope to obtain the greatest cost saving. When the critical activity is stretched and the duration of project completion is extended, other paths may also become noncritical, therefore, the noncritical activities can be stretched again until all the slack in the different noncritical paths network is used up.

Algorithm of SNCA:

Step 1: Draw the network project

Step 2: Determine the normal time and normal cost for each activity to determine critical and noncritical activities

Step 3: Compute the normal total cost and normal duration of the project completion. If F equals the normal duration of completion then we stop the procedure

Step 4: Determine the crash time and crash cost for each activity to compute the cost slope

Step 5: Crash all activities in the project simultaneously then determine the critical path and noncritical paths. Also, identify the critical activities

Step 6: Compute the new total cost by adding the cost of the crashing to the current total cost.

Step 7: Stretch noncritical activities: Start with those noncritical activities that will yield the greatest savings-those with the greatest cost slope. The noncritical activities can be stretched up to their normal time until all the slack in the different noncritical paths network is used up and then the saving cost of stretching all noncritical activities is found

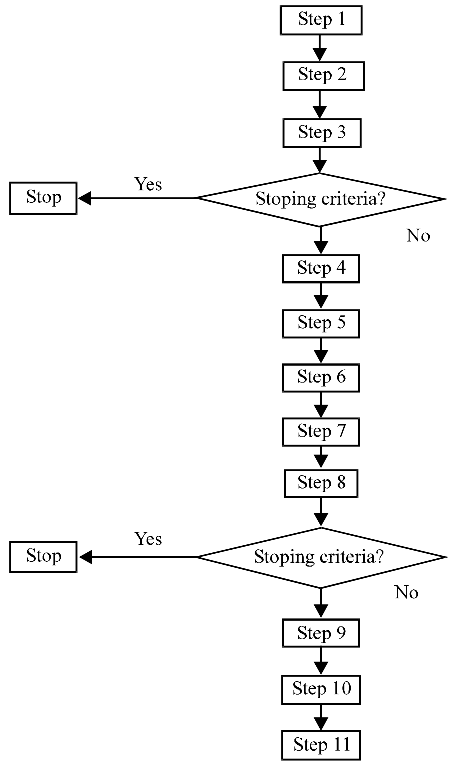

Fig. 1: Algorithm of stretching noncritical and critical activities

Step 8: The total cost of project completion in T is computed by subtracting the cost savings of stretching all noncritical activities from the cost of crashing all activities. If F = T then we stop the procedure (Fig. 1)

Step 9: Stretch critical activities (in critical path) for meeting F: Start with the critical activities that will yield the greatest savings-with the greatest cost slope. The critical activities can be stretched up to their normal time

Step 10: Stretch noncritical activities again: When critical activity is stretched and the duration of project completion is extended, other paths may also become noncritical, therefore, the noncritical activities can be stretched over again until all the slack in the different noncritical paths is used up. Then find the saving cost of stretching critical activities and noncritical activities. This gives the optimum (least cost) schedule called optimum duration

Step 11: Find the total cost for meeting F by subtracting summation saving cost of stretching noncritical activities and summation saving cost of stretching critical activities from the initial cost of crashing all activities

Practical examples: This study has been applied on two construction projects where data were taken from the 7 Nissan General Company in Iraq, the company is specialized in designing and execution of Bridges, Housing complexes, Concrete towers, Commercial and Industrial Buildings, Seeds Silos, Water and Water Treatment, Plants, Dams, ... , Etc. (Table 1).

Table 1: Information about the projects

Project name	Project type	Default start date for the project	The expected date of completion of the project in normal conditions
Project 1 (Al-Saidia)	House construction	1-1-2011	30 weeks
Project 2 (Al-Badia)	Plant construction	20-1-2011	27 weeks
Project 3(Al-Bounok)	House construction	10-2-2011	29weeks
Project 4 (Al-Salam)	Plant construction	1-1-2011	80 weeks
Project 5 (Al-Karama)	Plant construction	11-3-2011	70 weeks

The company accepted to construct new projects. The maximum budgeting available for the client is \$B. The client desires to complete the building within T. Because of various circumstances he is forced to extend the completion of the project (meet F) to reduce the total cost. The following terms need to illustrate.

A_i : Project's activities, where i = (1,2,... , n)
F : The desired project completion time at least total cost
B : Maximum available budget
T : Shortest possible duration to complete the project at least cost within the maximum available budget
$T_{N, I}$: Normal time for activity i
$T_{C, I}$: Crash time for activity i
$C_{N, I}$: Normal cost for activity i
$C_{C, I}$: Crash cost for activity i
U_i : Cost slope for activity i
CP : Critical path (longest path in the project network)
$D_{N, q}$: Normal time of critical activity q, where q = (1, 2, 3,... , L)
$D_{C, q}$: Crash time of critical activity q
$C_{N, j}$: Normal cost for noncritical activity j, where j = (1, 2, 3,... , m)
$C_{C, q}$: Crash cost for critical activity q
U_C : Cost slope for critical activity C, where C begin with critical activity that has the biggest cost slope = (1, 2, 3,... , P)
U_S : Cost slope for critical activity S, where S begin with critical activity that has a smallest cost slope = (1,2,3,... ,Y)
$D_{r, S}$: Max reduction in duration of critical activity S
$T.C_C$: Total cost to complete the project by crashing critical activities
$T.C_E$: The Extra cost that adding to crash critical activities
$T.C.S_{c.R,P}$: Total cost for meeting F considering R_T and P_T
CCA : Crashing Critical Activities
U_x : Cost slope for noncritical activity x, where x begin with noncritical activity that has the biggest cost slope = (1, 2, 3,... , z)
$D_{r, C}$: Max stretching in duration of critical activity C
$D_{r, x}$: Max stretching in duration of noncritical activity x
$T.C_N$: Total cost to complete the project in normal condition
$T.C_a$: Total cost to complete the project by crashing all activities (crash condition)
$T.S.C_{nc}$: Saving cost obtained by stretching all noncritical activities to complete the project in T
$T.C.S_{nc}$: Total cost to complete the project by stretching noncritical activities

$T.S.C_c$	:	Saving cost of meeting the desired project completion time by stretching the critical activities with extending the project duration
$T.S.C_{na}$	:	Saving cost of stretching noncritical activities again if possible until all the slack in the different noncritical paths is used up
$T.C.S_c$	:	Total cost for meeting F
SNCA	:	Stretching Noncritical and Critical Activities
$\theta_{CrashForT}$	:	No. of steps to meet F or T considering F>T or F=T by crashing critical activities
$\theta_{StretchF}$	:	No. of steps to meet F considering F>T by stretching noncritical activities then critical activities. These steps are summation for three stages
$\theta_{StretchT}$	:	No. of steps by stretching noncritical activities
$\theta_{Stretch.c}$	:	No. of steps by stretching critical activities
$\theta_{Stretch\ na}$	:	No. of steps by stretching noncritical activities over again

General descriptions and formulations:

Cost slope for any activity.

$$\text{Cost slope} = \frac{C_c - C_n}{T_n - T_c} \tag{1}$$

Critical path for the project network in normal condition.

$$C.P._{\cdot K} = \sum_{q=1}^{L} D_{N.q} \tag{2}$$

Total cost of the project in normal condition.

$$T.C_N = \sum_{i=1}^{n} C_{N.i} \tag{3}$$

Total cost of the project by crashing all activities.

$$T.C_a = \sum_{i=1}^{n} C_{c.i} \tag{4}$$

Critical path for the project network by crashing all activities.

$$T \text{ or } C.P._{\cdot C} = \sum_{q=1}^{L} D_{c.q} \tag{5}$$

Saving cost obtained by stretching all noncritical activities.

$$T.S.C_n = \sum_{x=1}^{Z} D_{r,x}.U_x \tag{6}$$

Total cost to complete the project in T by stretching noncritical activities

$$T.C.S_n = \sum_{i=1}^{n} C_{c,i} - \sum_{x=1}^{Z} D_{r,x}.U_x \tag{7}$$

Savings cost obtained by stretching critical activities for meeting F.

$$T.S.C_c = \sum_{C=1}^{P} D_{r,c}.U_c \tag{8}$$

Saving cost obtained by stretching noncritical activities again if possible, without extending the project duration.

$$T.S.C_{n.a} = \left(\sum_{x=1}^{Z} D_{r,x}.U_x \right) \tag{9}$$

Total cost for meeting F by stretching noncritical and critical activities.

$$T.C.S_c = T.C_a\ [T.S.C_n + T.S.C_{n.a} + T.S.C_c\ ] \tag{10}$$

Total cost to for meeting F by stretching noncritical and critical activities considering R_T and P_T.

$$T.C.S_{c.R,P} = P_T - R_T + T.C_a\ [\ T.S.C_n + T.S.C_{n.a} + T.S.C_c] \tag{11}$$

The Extra cost that adding to crash critical activities.

$$T.C_E = \sum_{S=1}^{Y} D_{r,S}.U_S \tag{12}$$

The total cost of the project by crashing critical activities.

$$T.C_C = \sum_{i=1}^{n} C_{N,i} + \sum_{S=1}^{Y} D_{r,S}.U_S \tag{13}$$

Number of steps to meet F (considering F>T) or to complete the project in T (considering F = T) by crashing critical activities.

$$\theta_{CrashForT} = \sum_{S=1}^{Y} D_{r,S} \tag{14}$$

Number of steps to complete the project in T by stretching noncritical activities.

$$\theta_{Stretch\,T} = \sum_{x=1}^{Z} D_{r,x} \tag{15}$$

Table 2: Activities data in normal and crash conditions for Project 1

Activity code	Activity predecessor	Normal time T_n	Crash times T_c	Normal cost C_n	Crash cost C_c	Max reduction in time	Cost slope
A		2	1	20,000	30,000	1	10,000
B	A	3	1	60,000	100,000	2	20,000
C	B	2	1	30,000	40,000	1	10,000
D	C	2	1	20,000	30,000	1	10,000
E	D	4	2	100,000	150,000	2	25,000
F	B	3	2	150,000	180,000	1	30,000
G	E, F	2	1	200,000	300,000	1	100,000
H	G	10	7	500,000	620,000	3	40,000
I	H	15	10	650,000	850,000	5	40,000
J	H	7	5	250,000	300,000	2	25,000
K	I	2	1	20,000	25,000	1	5,000
L	J, K	9	6	300,000	420,000	3	40,000
M	H	2	1	20,000	25,000	1	5,000
N	H	3	2	30,000	40,000	1	10,000
O	H	6	4	120,000	150,000	2	15,000
P	I	7	4	450,000	570,000	3	40,000
Q	L, M	4	2	350,000	500,000	2	75,000
R	N, Q	6	3	550,000	760,000	3	70,000
S	O, R	7	5	450,000	600,000	2	75,000
T	J, P	5	3	350,000	450,000	2	50,000
U	S	5	3	250,000	320,000	2	35,000
V	T	4	2	150,000	220,000	2	35,000
W	U, V	4	2	100,000	150,000	2	25,000
Total cost				5,120,000	6,830,000		

Number of steps to meet F considering F>T by stretching noncritical then critical activities.

$$\theta_{\text{Stretch F}} = \sum_{x=1}^{Z} D_{r,x} + \sum_{C=1}^{P} D_{r,C} + \left(\sum_{x=1}^{Z} (D_{r,x}) \right) \tag{16}$$

Numerical data: In this study we intend to take two different projects to clarify the precise details and relative importance using the algorithm method. Table 2 and 3, summarize projects 1 and 2 respectively, with the addition of some data as shown in Table 4.

RESULTS AND DISCUSSION

Project completion in normal duration and normal cost: After determining the normal time and cost for each activity to determine the critical and noncritical activities. We can compute the critical path and total normal cost for construction projects 1 and 2 from Eq. 2 and 3, respectively.

Project 1:

$$C.P._{K} = \sum_{q=A}^{W} D_{N,q} = D_{N,A} + D_{N,B} + ...C_{N,U} + D_{N,W} = 27\text{(weeks)}$$

$$T.C_{N} = \sum_{i=A}^{W} C_{N,i} = C_{N,A} + C_{N,B} + ... + C_{N,V} + D_{N,W} = \$57,000$$

Table 3: Activities data in normal and crash condition for Project 2

Activities	Predecessors (precedence)	Normal time	Crash time	Normal cost	Crash cost	Max reduction in time	Cost slope
A_1	----	3	2	5,000	7,000	1	2,000
A_2	A_1	4	2	4,000	5,000	2	500
A_3	A_2	4	4	7,000	7,000	0	----
A_4	A_2	3	1	3,000	5,000	2	1,000
A_5	A_2	5	2	6,000	10,500	3	1,500
A_6	A_5 , A_3	4	3	8,000	10,000	1	2,000
A_7	A_4	3	1	4,000	5,500	2	750
A_8	A_7	6	4	6,000	9,000	2	1,500
A_9	A_6	7	4	5,000	8,000	3	1,000
A_{10}	A_8, A_9	4	2	6,000	7,500	2	750
A_{11}	A_3, A_5	9	7	3,000	4,000	2	500
Total cost in normal and crash conditions				\$57,000	\$78,500		

Table 4: Additional information concerning the projects under study

Projects	B	T	F
1	78,500	17	20
2	6,830,000	46	50

Project 2:

$$C.P._{K} = \sum_{q=A}^{W} D_{N,\,q} = D_{N,\,A} + D_{N,\,B} + \ldots C_{N,\,U} + D_{N,\,W} = 77(\text{weeks})$$

$$C.P._{K} = \sum_{i=A}^{W} C_{N,\,i} = C_{N,\,A} + C_{N,\,B} + \ldots + C_{N,\,V} + D_{N,\,W} = \$5{,}120{,}000$$

Project completion by crashing all activities simultaneously: We can find the shortest possible duration and the total cost for completion of projects 1 and 2 from the Eq. 5 and 4, respectively.

Project 1:

$$C.P._{C}(T) = \sum_{q=A}^{W} D_{C,\,q} = D_{C,\,A} + D_{C,\,B} + \ldots + C_{C,\,U} + D_{C,\,W} = 17(\text{weeks})$$

$$T.C_{a}(T) = \sum_{i=A}^{W} C_{C,\,i} = C_{C,\,A} + C_{C,\,B} + C_{C,\,C} + \ldots + C_{C,\,W} = \$78{,}500$$

Project 2:

$$C.P._{C}(T) = \sum_{q=A}^{W} D_{C,\,q} = D_{C,\,A} + D_{C,\,B} + \ldots + C_{C,\,U} + D_{C,\,W} = 46(\text{weeks})$$

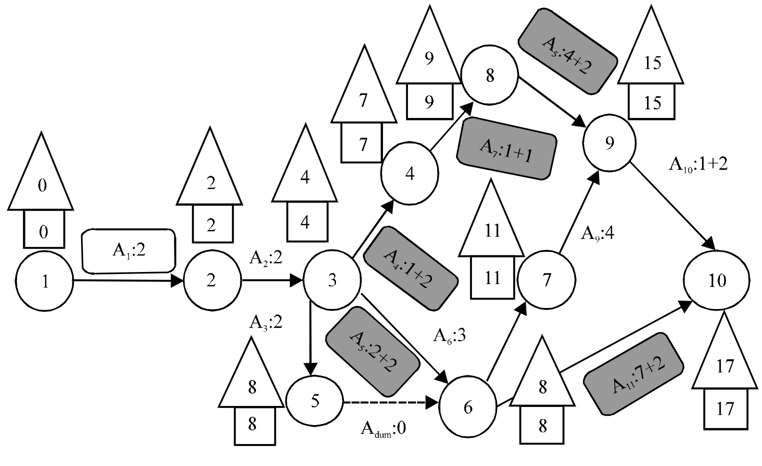

Fig. 2: Mechanism of SNA (as presented in grey rectangles) even all paths are become critical

$$\text{T.C.}_{\cdot a} = \sum_{i=A}^{W} C_{C,\,i} = C_{C,\,A} + C_{C,\,B} + C_{C,\,C} + \ldots + C_{C,\,W} = \$6{,}830{,}000$$

Completing project 1 within t by stretching noncritical activities: Crashing all activities simultaneously yields a project duration of 17 weeks and the expense of crashing all activities is \$78,500. The critical path: $A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow A_{DUM} \rightarrow A_6 \rightarrow A_9 \rightarrow A_{10}$ is the longest path; other paths are of shorter duration and consequently have no influence on the project duration. Thus, it is possible to stretch any noncritical activity by a certain amount without extending the project. Therefore, we can reduce the cost of crashing all the activities in the network project by stretching the noncritical jobs. We can begin with noncritical activity A_5 as it has the greatest cost slope (\$1,500) and can be stretched by up to 2 weeks in each path involving A_5 without extending the project duration and reducing the total cost (\$3,000), as well as noncritical activity A_8 which has a cost slope (\$15,00). This activity can be stretched by up to 2 weeks (bringing it to the normal time of 6 weeks) without extending the project duration and reducing total cost (\$3,000), etc. (Fig. 2 and Table 5).

We can find the cost saving obtained by stretching all noncritical activities to complete the project in T from Eq. 6 as follows:

$$\text{T.S.C}_n = \sum_{x=As}^{Al_1} D_{r,x}.U_x = D_{r,As}.U_{As} + D_{r,As}.U_{As} + \ldots + D_{r,Al_1}.U_{Al_1} = \$9{,}750$$

The final project cost is computed by subtracting the savings obtained by stretching all noncritical activity: (A_5) 2 weeks, (A_8) 2 weeks, (A_4) 2 weeks, (A_7) 1week, (A_{11}) 2 weeks from the initial crash cost: \$78,500-2(1,500)-2(1,500)-2(1,000)-1(750)-2(500) = \$68, 750. Similarly, the total cost to complete the project in T by stretching noncritical activities can be computed from Eq .7 as follows:

$$\text{T.C.S}_n = \sum_{x=A_1}^{Al_1} C_{C,i} - \sum_{x=As}^{Al_1} D_{r,x}.U_x = \left(C_{C.A_1} + C_{C.A_2} + C_{C.A_2} + \ldots + C_{C.Al_1}\right) - \left(D_{r.A5}.U_{A5} + D_{r.A8}.U_{A8} + .. + D_{r.A11}.U_{A11}\right)$$
$$= \$9{,}750 = \$68{,}750$$

Table 5: Mechanism of stretching noncritical activities for Project 2

Non critical activity that has a greatest slope respectively	Max increasing in time	Greatest Cost slope respectively	Length of Paths					Total cost aftersubtracting saving cost
			A_1	A_1	A_1	A_1	A_1	
			A_2	A_2	A_2	A_2	A_2	
			A_5	A_3	A_5	A_4	A_3	
			A_{11}	A_{DUM}	A_6	A_7	A_{DUM}	
				A_{11}	A_9	A_8	A_6	
					A_{10}	A_{10}	A_9	
							A_{10}	
			13	15	15	12	17	78,500
A_5	2	1,500	15	15	17	12	17	75,500
A_8	2	1,500	15	15	17	14	17	72,500
A_4	2	1,000	15	15	17	16	17	70,500
A_7	1	750	15	15	17	17	17	69,750
A_{11}	2	500	17	17	17	17	17	68,750
$T.S.C_{nc}$ =		\$9,750						

Completing project 2 within t by stretching noncritical activities: We can find the total savings cost by stretching all noncritical activities (Table 6) without extending the project duration from Eq. 6 as follows:

$$T.S.C_n = \sum_{x=T}^{M} D_{r,x}.U_x = D_{r,T}.U_T + D_{r,P}.U_P + ... + D_{r,M}.U_M = \$415,000$$

The total cost to complete the project in T by stretching noncritical activities can be computed from Eq. 7 as follows:

$$T.C.S_n = \sum_{i=A}^{w} C_{C,i} - \sum_{x=T}^{M} D_{r,x}.U_x = (C_{C,A} + C_{C,B} + C_{C,C} + ... + C_{C,W})$$
$$-(D_{r,T}.U_T + D_{r,P}.U_P + ... + D_{r,M}.U_M) = \$6,830,000 - \$415,5000 = \$6,415,000$$

Completing Project 1 to meet F: Reducing an activity's time from the normal time increases its cost, so stretching time from the crash time reduces its cost. To meet F (the desired project completion time at least total cost), we begin to stretch the critical activity to its normal time for increasing the project completion time. Stretching the critical activity that has the biggest cost slope obtains the greatest cost saving. We can begin with critical activity A_6 as it has the greatest cost slope (\$2,000) and can be stretched by up to 1week for each path involving A_6, thereby extending the project duration to 18 weeks and reducing the total cost (\$2,000) (Table 6). This last step stretches A_6 to 1 week, its stretch time, so no further stretching can be made to A_6. If necessary, an additional week can be stretched from another activity on the critical path that has the greatest cost slope, thus, the second step stretches A_1 to 1 week thereby extending the project duration to 19 weeks and reducing (\$2,000) from the project cost,. Any further stretching in project duration must be stretched from activity A_5 (third step) because it has the greatest cost slope (\$1,500) and can be stretched by up to 1week in each path involving A_5 to extend the project duration to meet F(20).

Table 6: Mechanism of stretching noncritical activities for Project 2

Noncritical activity that has a greatest slope respectively	Max increasing in time	Greatest cost slope respectively	Length of path														Total cost aftersubtracting
			A B C D E G H O S U W	A B C D E G H N R S U W	A B C D E G H M Q R S U W	A B C D E G H I P T V W	A B C D E G H I K L Q R S U W	A B C D E G H J L Q R S U `W	A B C D E G H J T V W	A B F G H O S U W	A B F G H N R S U W	A B F G H M Q R S U W	A B F G H I P T V W	A B F G H I K L Q R S U W	A B F G H J L Q R S U W	A B F G H J T V W	
			28	29	30	35	46	40	26	26	27	28	33	44	38	24	6,830,000
T	2	50000	28	29	30	37	46	40	28	26	27	28	35	44	38	26	6,730,000
P	3	40000	28	29	30	40	46	40	28	26	27	28	38	44	38	26	6,610,000
V	2	35000	28	29	30	42	46	40	30	26	27	28	40	44	38	28	6,540,000
F	1	30000	28	29	30	42	46	40	30	27	28	29	41	45	39	29	6,510,000
J	2	25000	28	29	30	42	46	42	32	27	28	29	41	45	41	31	6,460,000
O	2	15000	30	29	30	42	46	42	32	29	28	29	41	45	41	31	6,430,000
N	1	10000	30	30	30	42	46	42	32	29	29	29	41	45	41	31	6,420,000
M	1	5000	30	30	31	42	46	42	32	29	29	30	41	45	41	31	6,415,000
$T.S.C_{nc}$ = \$415000																	

We can find the total saving cost $T.S.C_c$ by stretching critical activities to their normal time to meet the desired project completion time from Eq. 8.

$$T.C.S_c = \sum_{x=A6}^{A5} D_{r,c}.U_C = D_{r,A6}.U_{A6} + D_{r,A1} + U_{A1} + D_{r,A5}.U_{A5} = 1(2,000) + 1(2,000) + 1(1,500) = \$5,500$$

When the critical activity is stretched and the duration of the project completion is extended, other paths may also become noncritical, therefore, the noncritical activities may be stretched again until all the slack in the different noncritical paths network is used up. Therefore, we can observe that the noncritical activity A_7 cannot be stretched to more than one week. Note that the normal time to finish it is 3 weeks. The reason being is that the existence of A_7 in the path only shortens the critical path by one week (Fig. 3).

So, we can see that the noncritical activity A_7 which has a cost slope (\$750) can be stretched by up to 1week in each path involving A_7, without extending the project duration and thereby reducing the total cost (\$750) (Table 7).

We can compute the cost savings $T.S.C_{n.a}$ obtained by stretching the noncritical activities again from Eq. 9.

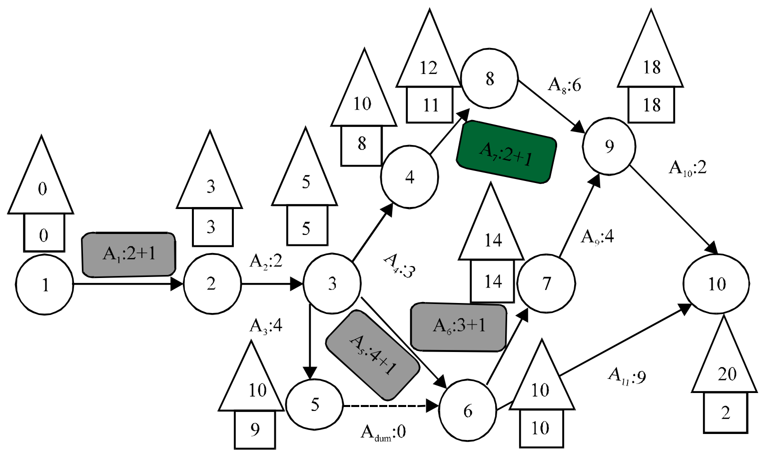

Fig. 3: Network of Project 1, grey rectangles refer to the stretching of critical activities and dark green rectangles indicate to the stretching noncritical activities again

Table 7: Mechanism of *SNCA* for Project 1

Critical activity that has a greatest cost slope respectively	Noncritical activity that has a greatest cost slope respectively *again*	Max increasing in time	Greatest cost slope respectively	Length of paths					Total cost after subtracting
				A_1	A_1	A_1	A_1	A_1	
				A_2	A_2	A_2	A_2	A_2	
				A_5	A_3	A_5	A_4	A_3	
				A_{11}	A_{dum}	A_6	A_7	A_{dum}	
					A_{11}	A_9	A_8	A_6	
						A_{10}	A_{10}	A_9	
								A_{10}	
				17	17	17	17	17	68,750
A_6		1	2,000	17	17	18	17	18	66,750
	A_7	1	750	17	17	18	18	18	66,000
A_1		1	2,000	18	18	19	19	19	64,000
A_5		1	1,500	19	18	20	19	19	62,500
T.C.S$_c$ = \$62,500									

$$T.C.S_{na} = \sum_{x=A7}^{A7} D_{r,x}.U_x = D_{r,A7}.U_{A7} = \$750$$

(Notice: it may not be necessary to repeat Eq. 9 again after stretching critical activities when all noncritical activities are stretched to their normal time (see Section Completing Project 2 to meet F)

Therefore, the total cost T.C.S$_c$ for meeting the desired project completion time can be computed by subtracting T.S.C$_n$, T.S.C$_c$ and T.S.C$_{n.a}$ from T.C$_a$. From Eq. 10 we can compute T.C.S$_c$ as follows:

$$T.C.S_c = \sum_{i=1}^{n} C_{C,i} - \left[\sum_{x=1}^{z} D_{r,x}.U_x + \left(\sum_{x=1}^{z} D_{r,x}.U_x\right) + \sum_{C=1}^{P} D_{r,C}.U_C\right] + \sum_{i=A1}^{A1_1} C_{C,i} - \sum_{x=A5}^{A1_1} D_{r,x}.U_x - \sum_{x=A7}^{A7} D_{r,x}.U_x - \sum_{C=A1}^{A5} D_{r,c}.U_C$$
$$= \$78{,}500 - \$9{,}750 - \#750 - \$5500 = \$62{,}500$$

Table 8: Mechanism of *SNCA* for Project 2

Critical activity that has a greatest cost slope respectively	Max increasing in time	Greatest Cost slope respectively	Length of Path														Total cost after subtracting
			A B C D E G H O S U W	A B C D E G H N R S U W	A B C D E G H M Q R S U W	A B C D E G H I P T V W	A B C D E G H I K L Q R S U W	A B C D E G H J L Q R S U W	A B C D E G H J T V W	A B F G H O S U W	A B F G H N R S U W	A B F G H M Q R S U W	A B F G H I P T V W	A B F G H I K L Q R S U W	A B F G H J L Q R S U W	A B F G H J T V W	
			30	30	31	42	46	42	32	29	29	30	41	45	41	31	6,415, 000
G	1	100,000	31	31	32	43	47	43	33	30	30	31	42	46	42	32	6,315,000
S	2	75,000	33	33	34	43	49	45	33	32	32	33	42	48	44	32	6,165, 000
Q	1	75,000	33	33	35	43	50	46	33	32	32	34	42	49	45	32	6,090, 000
$T.C.S_c$ = $325,000																	

Completing Project 2 to meet F: To meet F, we begin to stretch the critical activity to its normal time. Stretching the critical activity that has the biggest cost slope obtains the greatest cost saving. Therefore, we can begin with critical activity G because it has the greatest cost slope ($100,000) and can be stretched by up to 1week in each path involving G to extend the project duration to meet F (50 weeks) and reducing the total cost $100,000 (Table 8).

Stretching G by 1 week extends the project duration to 47 weeks and reduces the project cost by $100,000 (the cost slope of G), bringing the project cost down to $6,415,000-$100,000 = $6,315,000. This step does not change the critical path as it is still the longest (47 weeks). Therefore, this last step stretches G to 1 week, its stretch time, so no further stretching can be made to G. If necessary, an additional week can be stretched from another activity on the critical path that has the next greatest cost slope. Thus the second step stretches S to 2 weeks, which extends the project duration to 49 weeks and reduces the project cost by $150,000 (the cost slope of S is $75,000), thereby bringing the project cost down to $6,315,000-$150,000 = $6,165,000. This step does not change the critical path, any further stretching in project duration must be stretched from activity Q (third step) as it has the greatest cost slope ($75,000) and can be stretched by up to 1week in each path involving S to extend the project duration to meet F (50 weeks) bringing the project cost down to $6,165,000 -$75,000 = $6,090,000

From Eq. 8 we can find the saving cost $T.S.C_c$ by stretching the critical activity to its normal.

$$T.S.C_c = \sum_{C=G}^{Q} D_{r,C}.U_C = D_{r,G}.U_G + D_{r,S}.U_S + D_{r,Q}.U_Q$$

$$= 1(100,000) + 1(75,000) + 1(75,000) + 1(75,000) + \$325,000$$

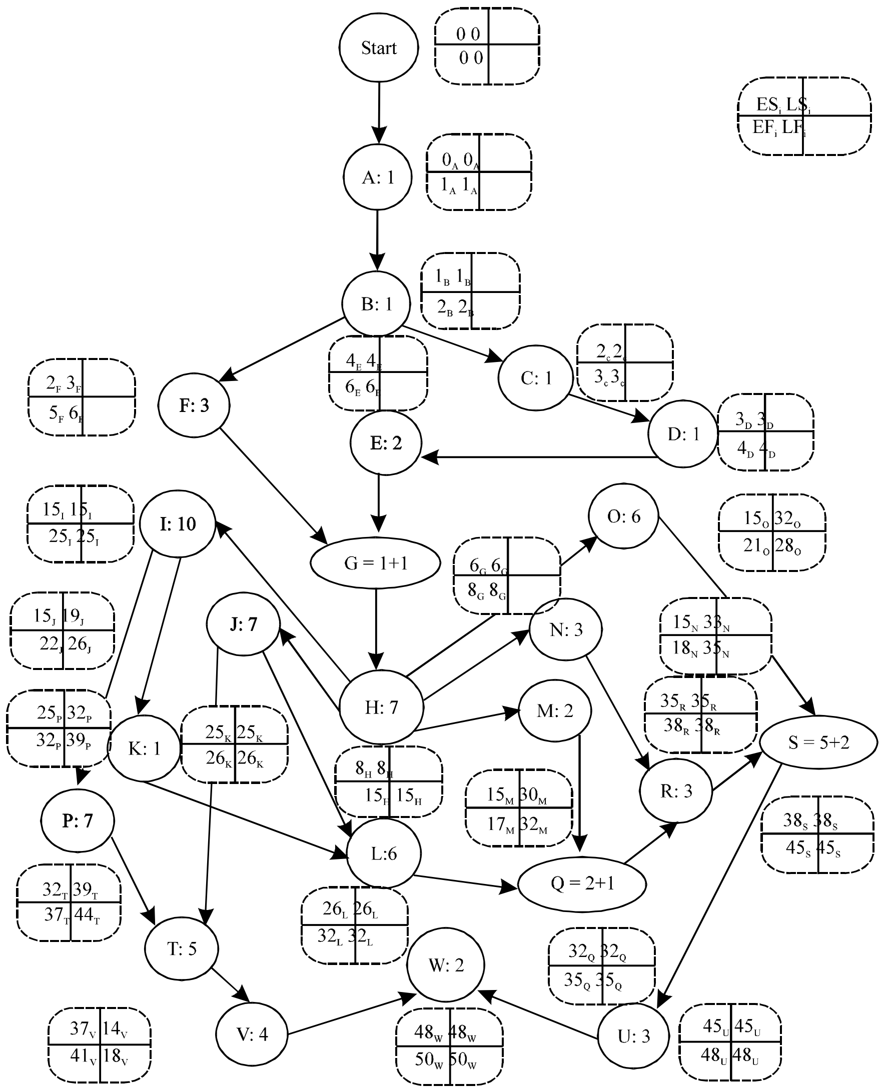

Fig. 4: Network of Project 4, red oval shape indicate to the stretching critical activities

We can observe in this case that no non-critical activities can be stretched again. Therefore, the cost saving $T.S.C_{n.a}$ obtained by stretching noncritical activities again from Eq. 9 is zero.

The total cost $T.C.S_c$ for meeting the desired project completion time computed by subtracting $T.S.C_n$, $T.S.C_c$ and from the $T.C_a$ from Eq. 10 can be computed by $T.C.S_c$ as follows:

$$
\begin{aligned}
T.C.S_c &= \sum_{i=1}^{n} C_{C,i} - \left[\sum_{x=1}^{z} D_{r,x}.U_x + \sum_{C=1}^{P} D_{r,C}.U_C\right] = \sum_{i=}^{w} C_{C,i} - \left[\sum_{x=T}^{M} D_{r,x}.U_x + \sum_{C=G}^{Q} D_{r,C}.U_C\right] \\
&= C_{C,A} + C_{C,B} + C_{C,C} + \ldots + C_{C,W} - [(D_{r,T} + U_T + D_{r,P} + U_P + \ldots + D_{r,M} + U_M) \\
&\quad +(D_{r,G} + U_G + D_{r,S} + U_S + D_{r,S} + U_S + D_{r,Q} + U_Q
\end{aligned}
$$

Discussing the mechanism of re-stretching noncritical activities: When the critical activity is stretched and the duration of project completion is extended, other paths may also become noncritical, therefore, the noncritical activities may be stretched again until all the slack in the different noncritical paths network is used up. Therefore, depending on the nature of the project network we can classify two cases in stretching noncritical activities again: The first case (Project 2) explains the difference in length between the critical path and the other paths. This difference is greater than or equal to the time of stretching the noncritical activities and is called the slack of the paths. We can reduce the slack in these paths (depending on the nature of the network project) by stretching all the noncritical activities from the crash conditions to normal conditions. Therefore, stretching critical activities 4 weeks to meet F (50 weeks) does not affect the other paths because they were already stretched to their normal time (Table 8 and Fig. 4).

The second case (Project 1) explains the difference between the critical path and the other paths, this difference is smaller than or equal to the time of stretching the noncritical activities. Therefore, we can use up all the slack in these paths by stretching most of the noncritical activities from crash conditions to normal conditions. Row 1 in table 6 shows that the difference between the critical path and other paths is used up after stretching all noncritical activities, therefore, stretching the critical path 3 weeks to meet F (20 weeks) affects the other paths because they were critical activities and some of them become noncritical.

CONCLUSION

In our practical project the SNCA approach has been presented in this study. At first this approach proposes a trade-off of time against cost to complete the project in T then a trade-off of cost against time to meet the project in F, if there is a difference between them.

The time-cost trade-off and cost-time tradeoff presented in SNCA provides us with a systematic and logical approach for decision making when the owners or project managers intend to extend the completion of the project to reduce the total cost for various circumstances that force them to do so.

The cost of the network activities has been optimized for various overall durations. The optimum trade-off of time against cost and vice versa has been made by SNCA. This approach is an acceptable tool of management and provides a superior approach for planning, scheduling and controlling project progress, as well as a very real and valuable asset to contractors in convincing the owner of their potential and abilities. With the introduction of better and more rigorous methods of planning work, together with cost analysis, the construction control will become more systematic. Based on our example, we can also conclude that in the SNCA approach:

- An activity can be stretched up to its normal time, which is assumed to be its least-costly time; extending the activity beyond the normal pace will not produce any additional savings and might well increase the cost
- It is not necessary to crash every activity to finish the project in T
- It is not necessary to use up all the slack in the different noncritical paths network when the noncritical activities are stretched to their normal time
- We obtain the cost saving by increasing the duration of noncritical activities that have the greatest cost slope to their normal time to use up all the slack in the different noncritical paths. Then we obtain the cost saving by increasing the duration of critical activities that have the greatest cost slope to their normal time

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