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Modeling Effects of Three Nano-scale Physical Phenomena on Instability Voltage of Multi-layer MEMS/NEMS: Material Size Dependency, van der Waals Force and Non-classic Support Conditions

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ABSTRACT

Although many micro/nano-electromechanical systems (MEMS/NEMS) are implemented by multi-layer components, only few researchers have modeled the instability of these multi-layer structures. Herein, the electrostatic instability of multi-layer MEMS/NEMS is modeled using a non-classic continuum mechanics theory. Three nano-scale physical phenomena which highly affect the pull-in performance of MEMS/NEMS including size dependency, van der Waals force and non-classic support conditions have been considered in the model for the first time. The modified couple stress theory is applied to examine the size effect on the instability of the MEMS/NEMS at submicron separations. The proposed model takes non-classic boundary conditions into account using translational and rotational springs at supported end of the MEMS/NEMS. In order to solve the constitutive equation of the MEMS/NEMS, modified Adomian decomposition method is employed as well as lumped parameter model and numerical method. The obtained analytical solution is more accurate than lumped model and agrees well with numerical one. The results reveal significant influence of the size dependency, elastic boundary condition and van der Waals attraction on the pull-in characteristics of MEMS/NEMS.

Key words: Multi-layer MEMS/NEMS, size effects, van der Waals force, non-classic boundary conditions, static pull-in instability, modified couple stress theory

INTRODUCTION

With recent developments in nanotechnology, micro/nano-beams have become one of the most important components in constructing micro/nano-electro-mechanical systems (MEMS/NEMS) (Ke and Espinosa, 2006; Osterberg, 1995; Gupta, 1997; Kuang and Chen, 2005; Mojahedi *et al.*, 2010). A beam-type MEMS/NEMS consists of two conducting electrodes; one is movable and another is fixed. The electrodes may be single-layer or multilayer. At a critical voltage/deflection which is known as pull-in voltage/deflection, the instability occurs and the movable electrode pulls-in onto the fixed one. Different analytical, numerical, finite element methods have been proposed to model the pull-in behavior of micro-beams (Das and Batra, 2009; Moghimi Zand and Ahmadian, 2007; Tajalli *et al.*, 2009; Pamidighantam *et al.*, 2002; Chowdhury *et al.*, 2005; Batra *et al.*, 2007; Lin and Zhao, 2008). Although, many electromechanical devices are implemented by multilayer membranes (Chang *et al.*, 2009; Rong *et al.*, 2004; Nie *et al.*, 2006, 2009), most of researchers focused on single-layer devices and only few (Moghimi Zand and Ahmadian, 2007; Rong *et al.*, 2004; Nie *et al.*, 2009) have modeled the pull-in performance of multi-layer structures. Moghimi Zand and Ahmadian (2007) investigated the pull-in behavior of multilayer micro-plates using finite element method. Rong *et al.* (2004) proposed an analytical model for pull-in behavior of multilayer micro-beams. Neglecting nano-scale effects is a common assumption in MEMS literature. However, with decrease in dimensions to nano-scale, many essential phenomena i.e., van der Waals (vdW) force and size dependency appear which should be considered in modeling NEMS. Furthermore, Boundary Conditions (B.C.) of real nano-structures are not always homogeneous (as usually assume in theoretical models) and may be flexible by rotation/translation due to limitations of manufacturing processes etc. In this study, three of these phenomena are considered for simulation of instability of multilayer beam-type MEMS/NEMS.

The first effect that appears at submicron-scale is the size dependency of material characteristics. Experiments (Fleck *et al.*, 1994; Lam *et al.*, 2003; McFarland and Colton, 2005) demonstrate that the size dependency is an inherent property of metals when the characteristic size of the structures is in the order of the internal material length scale. It is found that torsional hardening of copper wire increases by a factor of 3 as the wire diameter decreases from 170 to 12 μm (Fleck *et al.*, 1994). For beam-type nanostructures, the characteristic size (usually beam thickness) is comparable with metal length scale parameter (Kong *et al.*, 2008). Therefore, size dependency must be considered in simulating these structures, especially in the case of multilayer beams with ultra-thin layers. From theoretical point of view, classical continuum mechanics is not capable of explaining the size effects. Recently, modified couple stress theory has been proposed by Yang *et al.* (2002) which can model size dependency of structures (Asghari *et al.*, 2010; Wang, 2009; Jomehzadeh *et al.*, 2011).

The second phenomenon that becomes important at submicron distances is the presence of vdW attraction. When the initial gap between the actuator components is typically below several ten nanometers, vdW force can highly influences the pull-in performance of actuator (Gusso and Delben, 2008). Spengen *et al.* (2002) studied the stiction in MEMS switches due to vdW force. Dequesnes *et al.* (2002) modeled the effect of vdW attraction on the instability of NEMS probe. Batra *et al.* (2009) studied the pull-in behavior of micro-plates in the presence of vdW force. The dynamic behavior of a nano-actuator was investigated by Lin and Zhao (2003) considering vdW force. Further information concerning the effect of vdW force on the pull-in instability of MEMS/NEMS actuators and its modeling can be found in other references (Rotkin, 2002; Wang *et al.*, 2004; Ramezani *et al.*, 2008; Ke *et al.*, 2005; Guo and Zhao, 2004; Lin and Zhao, 2007; Moghimi Zand and Ahmadian, 2010).

Finally, the third effect that must be considered in modeling ultra-small structures is the characterization of real B.C. Characterizing B.C. is important in flexible optical waveguides (Ollier, 2002), atomic force microscope probes (Pedersen, 2000), micro/nano-bridges (Michael and Kwok, 2007) and MEMS/NEMS switches (Nie *et al.*, 2009). According to limitations of manufacturing techniques in micro/nano-scale productions, any ideal B.C. such as clamped condition could not be acceptable and the boundary support conditions need to be theoretically quantified and experimentally validated (Rinaldi *et al.*, 2008). Rinaldi *et al.* (2007) characterized the B.C. of micro-cantilevers through electromechanical test. Yunqiang *et al.* (2008) studied the B.C. effect on the mechanical responses of micro-plates. Real B.C. of structures is usually modeled using artificial springs at the supported end.

Unfortunately, pull-in performance of multi-layer MEMS/NEMS has not been addressed well in the literature. Furthermore, to the knowledge of the authors, none of the three mentioned phenomena have contributed together in any of the pull-in models already proposed. In this study, pull-in performance of multilayer MEMS/NEMS is investigated for the first time considering the effects of vdW force, size dependency and double-spring supported B.C. together. In order to solve the nonlinear constitutive equation of nano-structure, analytical method is utilized as well as lumped model and numerical method.

MODIFIED COUPLE STRESS THEORY AND GOVERNING EQUATIONS

Figure 1 show simple beam model of multi-layer MEMS/NEMS. The cantilever MEMS/NEMS is modeled by an n-layer Euler-Bernoulli beam of length L with a uniform rectangular cross section of width b and varied thickness from h_1 to h_n which are suspended over a conductive substrate. Only the top layer of the beam is temporarily assumed to be a conductor, while other left layers are all dielectric. The total thickness of dielectric layers is H. Young's and shear modulus of i^{th} layer are E_i and μ_i, respectively. The relative permittivity of the i^{th} layer is ε_i. The multi-layer beam is deformable while the conductive substrate is rigid. The artificial rotational and translation springs with stiffness of K_u and K_θ are used to model the real B.C. of supported end of the actuator. In order to develop the governing equation of the static beams, modified couple stress theory is applied. Based on this theory, the strain energy density is written as Yang *et al.* (2002):

$$\bar{u}=\frac{1}{2}(\sigma:\varepsilon+m:\chi) \tag{1}$$

where, the stress tensor σ, strain tensor ε, deviatoric part of the couple stress tensor m and symmetric curvature tensor χ are defined by the following:

$$\sigma=\lambda \mathrm{tr}(\varepsilon)I+2\mu\varepsilon \tag{2a}$$

$$\varepsilon=\frac{1}{2}\left((\Delta u)+(\Delta u)^T\right) \tag{2b}$$

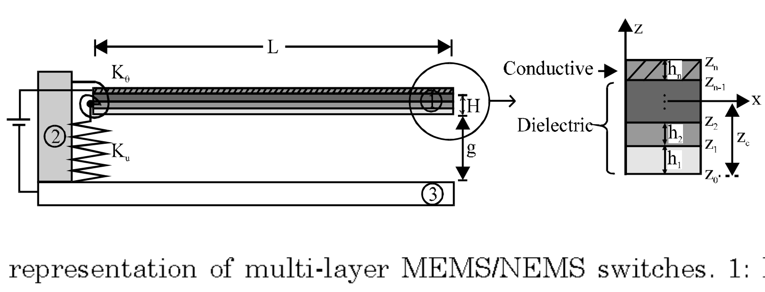

Fig. 1: Schematic representation of multi-layer MEMS/NEMS switches. 1: Beam, 2: Dielectric spacer and 3: Fixed ground plane

$$m = 2l^2 \mu\chi \tag{2c}$$

$$\chi = \frac{1}{2}\left((\Delta\theta) + (\Delta\theta)^T\right) \tag{2d}$$

$$\theta = \frac{1}{2}\nabla \times u \tag{2e}$$

where, λ, μ and l are Lame constant, shear modulus and the material length scale parameter, respectively. In above relations, θ denote rotation vector. According to the basic hypotheses of Euler-Bernoulli beams in one dimension, the displacement field is assumed as (Dym and Shames, 1984):

$$u = -z\frac{\partial w(X)}{\partial X}, v = 0, w = w(X) \tag{3}$$

where, u, v and w represent the displacement along X, Y and Z axes, respectively. By neglecting the Poisson's effect (to facilitate the formulation), considering small elastic deformation and substituting relations (2 and 3) in Eq. 1, integrating, adding the elastic energy of the springs and considering the work done by external forces, the energy of the system can be written as:

$$U = \frac{1}{2}\int_0^L \int_A \left\{E(z)z^2\left(\frac{\partial^2 w}{\partial X^2}\right)^2 + \mu l^2(z)\left(\frac{\partial^2 w}{\partial X^2}\right)^2\right\} dA dX + \frac{1}{2}K_u\left(\frac{\partial w(0)}{\partial X}\right)^2 \frac{1}{2}K_u\left(w(0)\right)^2 \tag{4a}$$

$$V = \int_0^L f_{else}(X)\, w(X)\, dX + \int_0^L fvdW(X)\, w(X)\, dX \tag{4b}$$

where, f_{elec} and f_{vdW} are the electrostatic and vdW forces per unit length of the beam, respectively. Note that in above equation, X-axis is the neutral axis where its distance from the bottom of the beam, Z_c, is determined in Appendix A. Note that the constitutive materials of the beam are assumed linear elastic and only the static deflection of the beam is considered.

Applying method described by Rong *et al.* (2004) in conjunction with the model presented by Gupta (1997), the electrostatic force enhanced with first order fringing correction can be derived as the following equation:

$$f_{else} = \frac{\varepsilon_0 b V^2\left[1 + \frac{0.65}{b}\left(g + H + \sum_{i=1}^{n-1} h_i/\varepsilon_i - w\right)\right]}{2\left(g + H + \sum_{i=1}^{n-1} h_i/\varepsilon_i - w\right)^2} \tag{5}$$

where, ε_0 = 8.854×10-12 $C^2N^{-1}\,m^{-2}$ is the permittivity of vacuum, V is the applied voltage and g is the initial gap between the movable and the ground electrode. For ultra-thin NEMS, the finite size and quantum effects must be considered when calculating the surface charge distribution especially for narrow beams.

In nano-scale separation regime (typically below several ten nanometers), the attraction between two surfaces is defined by vdW force. Using the model presented in (Dequesnes *et al.*, 2002), vdW force per unit length of the multi-layer beam (f_{vdW}) can be derived as:

$$f_{vdW}=\sum_{i=1}^{n}\frac{A_i b}{6\pi}\left[\frac{1}{(g+\sum_{k=1}^{i-1}h_k-w)^3}-\frac{1}{(g+\sum_{k=1}^{i}h_k-w)^3}\right] \tag{6}$$

where, A_i is the Hamaker constant corresponds to ith layer/ground interaction.

Now, by applying Hamilton principle, i.e., δ(V-U) = 0 where δ denotes the variation symbol, the governing equilibrium of the NEMS is derived from Eq. 5 as:

$$\left\{\frac{b}{3}\sum_{i=1}^{n}E_i\left(z_i^3-z_{i-1}^3\right)+b\sum_{i=1}^{n}\mu_i l_i^2(z_i-z_{i-1})\right\}\frac{d^4w}{dX^4}=$$
$$\frac{\varepsilon_0 bV^2\left[1+\frac{0.65}{b}(g+H\sum_{i=1}^{n=1}h_i/\varepsilon_i-w)\right]}{2\left(g+H\sum_{i=1}^{n=1}h_i/\varepsilon_i-w\right)^2}+\sum_{i=1}^{n}\frac{A_i b}{6\pi}\left[\frac{1}{(g+\sum_{k=1}^{i=1}h_k-w)^3}-\frac{1}{(g+\sum_{k=1}^{i}h_k-w)^3}\right] \tag{7a}$$

$$\left\{\frac{b}{3}\sum_{i=1}^{n}E_i\left(z_i^3-z_{i-1}^3\right)+b\sum_{i=1}^{n}\mu_i l_i^2(z_i-z_{i-1})\right\}\frac{d^3w(0)}{dX^3}=K_u\,w(0) \tag{7b}$$

$$\left\{\frac{b}{3}\sum_{i=1}^{n}E_i\left(z_i^3-z_{i-1}^3\right)+b\sum_{i=1}^{n}\mu_i l_i^2(z_i-z_{i-1})\right\}\frac{d^2w(0)}{dX^2}=K_\theta\frac{dw(0)}{dX} \tag{7c}$$

$$\frac{d^3w(L)}{dX^3}=\frac{d^2w(L)}{dX^2}=0 \tag{7d}$$

Note that Eq. 7a and b reveal the force and moment balance at the supported end of the nano-beam and Eq. 7d implies zero force and moment at the free end. Special cases, i.e., cantilever beam, can be modeled easily by setting spring stiffness as infinite ($K_u = K_\theta = 8$).

Using the substitutions, $\hat{w} = w/g$ and $x = X/L$, Eq. 7 transform into:

$$\frac{d^4\hat{w}}{dx^4}=\frac{\beta}{(1+\eta)\left(\bar{g}-\hat{w}(x)^2\right)}+\frac{\gamma\beta}{(1+\eta)\left(\bar{g}-\hat{w}(x)\right)}+\frac{1}{1+\eta}\sum_{i=1}^{n}\left[\frac{\alpha_i}{\left(\hat{g}_{i=1}-\hat{w}(x)^3\right)}-\frac{\alpha_i}{\left(\hat{g}_i-\hat{w}(x)^3\right)}\right] \tag{8a}$$

$$\hat{w}'''(0)=\bar{K}_u\hat{w}(0) \tag{8b}$$

$$\hat{w}''(0)=\bar{K}_\theta \hat{w}'(0) \tag{8c}$$

$$\hat{w}'''(1)=\hat{w}^{11}(1)=0 \tag{8d}$$

In above equations, the non-dimensional parameters are defined as:

$$\alpha_i=\frac{A_i L^4}{2\pi g^4 \sum_{i=1}^{n} E_i\left(z_i^3-z_{i-1}^3\right)} \tag{9a}$$

$$\beta=\frac{3\varepsilon_0 V^2 L^4}{2g^3 \sum_{i=1}^{n} E_i\left(z_i^3-z_{i-1}^3\right)} \tag{9b}$$

$$\gamma=0.65\frac{g}{b} \tag{9c}$$

$$\eta=\frac{3\sum_{i=1}^{n}\mu_i l_i^2 (z_i-z_{i-1})}{\sum_{i=1}^{n} E_i\left(z_i^3-z_{i-1}^3\right)} \tag{9d}$$

$$\bar{K}_u=\frac{K_u L^3}{\frac{b}{2}\sum_{n=1}^{n} E_i\left(z_i^3-z_{i-1}^3\right)+b\sum_{i=1}^{n}\mu_i l_i^2 (z_i-z_{i-1})} \tag{9e}$$

$$\bar{K}_\theta=\frac{K_\theta L}{\frac{b}{3}\sum_{n=1}^{n} E_i\left(z_i^3-z_{i-1}^3\right)+b\sum_{i=1}^{n}\mu_i l_i^2 (z_i-z_{i-1})} \tag{9f}$$

$$\bar{g}\simeq 1+\frac{H}{g}+\frac{1}{g}\sum_{i=1}^{n-1}\frac{h_i}{\varepsilon_i} \tag{9g}$$

$$\hat{g}_i=1+\frac{1}{g}\sum_{k=1}^{i} h_k \tag{9h}$$

Relations (8) and (9) present the governing equation of multi-layer nanostructures.

SOLVING METHODS

Lumped parameter model: The lumped model proposed by Osterberg (1995) for single-layer MEMS without considering nano-scale effects. Based on his work, a lumped parameter model can be developed from Eq. 8 taking into account the size dependency, vdW attraction and non-classic B.C. neglected in his investigation. The lumped parameter model only simulates the tip of the beam;

therefore in Eq. 8, $\hat{w}$ (x) is identical to $\hat{w}_{tip}$. Using the same notation defined in the previous sections and replacing $\hat{w}$ (x) with $\hat{w}_{tip}$, the pull-in voltage of the lumped parameter nano-structure model is obtained as (See Appendix B):

$$\beta = \frac{8(1+\eta)\left(1+4[2\bar{K}_u^{-1}+\bar{K}_\theta^{-1}]\right)^{-1}\hat{w}_{tip} - \sum_{i=1}^{n}\left[\alpha_i\left(\hat{g}_{i-1}-\hat{w}_{tip}\right)^{-3} - \alpha_i\left(\hat{g}_{i-1}-\hat{w}_{tip}\right)^{-3}\right]}{1+\gamma\left(\bar{g}-\hat{w}_{tip}\right)}\left(\bar{g}-\hat{w}_{tip}\right)^2 \quad (10)$$

For any given α, η, K_θ, K_u and γ, the pull-in voltage and pull-in deflection can be obtained from Eq. 10 by setting $d\beta/d\hat{w}_{tip} = 0$ or plotting β vs. $\hat{w}_{tip}$ as another simple approach.

Analytical solution: Using Modified Adomian Decomposition (MAD) method (Kuang and Chen, 2005), the deflection of nano-beam in Eq. 8 can be represented as (see Appendix C):

$$w(x) = \sum_{n=0}^{\infty} w_n(x) = C_1 + C_2x + \frac{1}{2!}C_3x^2 + \frac{1}{3!}C_4x^3 + \int_0^x\int_0^x\int_0^x\int_0^x\left[\sum_{n=0}^{\infty} N_n(x)\right]dx\,dxdxdx \quad (11)$$

In above equation constants C_1-C_4 can be determined by solving the resulted algebraic equation from B.C. i.e., Eq. 8b-8d. The series N_n (x) represents Adomian polynomials which linearized the non-linear right hand side of Eq. 8a (Kuang and Chen, 2005). Using a recursive procedure and applying B.C., the solution of Eq. 8 is easily obtained. Details of the solution method can be found in Appendix C.

Numerical solution: In order to study the pull-in behavior of nanostructures, the boundary value problem (Eq. 8) is solved numerically using MAPLE commercial software. The step size of the parameter variation is selected based on the sensitivity of the parameter to the tip deflection. Furthermore, the numerical results are compared with those of analytical MAD solutions as well as lumped parameter model in the following section.

CASE STUDIES

Here, the capability of present model for simulating the pull-in phenomena in electromechanical systems is demonstrated. Two cases including multilayer micro-actuator and single-layer nano-actuator are investigated.

It was shown that by using six terms series, the global error between analytical and numerical results is within the acceptable range for engineering applications. Therefore, six terms are selected in the following section for the convenience of calculations and acceptable error. Higher accuracy can be obtained by evaluating more terms of the solution $\hat{w}$ (x).

Multilayer micro-actuator: Neglecting intermolecular forces is a common practice in MEMS literature. Omitting vdW force, relation (8a) is simplified and rewrote as:

$$\frac{d^4}{dx^4}[\hat{w}/\bar{g}] = \frac{\left[\beta/\bar{g}^3\right]}{(1+\eta)\left(1-[\hat{w}/\bar{g}]\right)^2} + \frac{(\gamma\bar{g})\left[\beta/\bar{g}^3\right]}{(1+\eta)\left(1-[\hat{w}/\bar{g}]\right)} \quad (12)$$

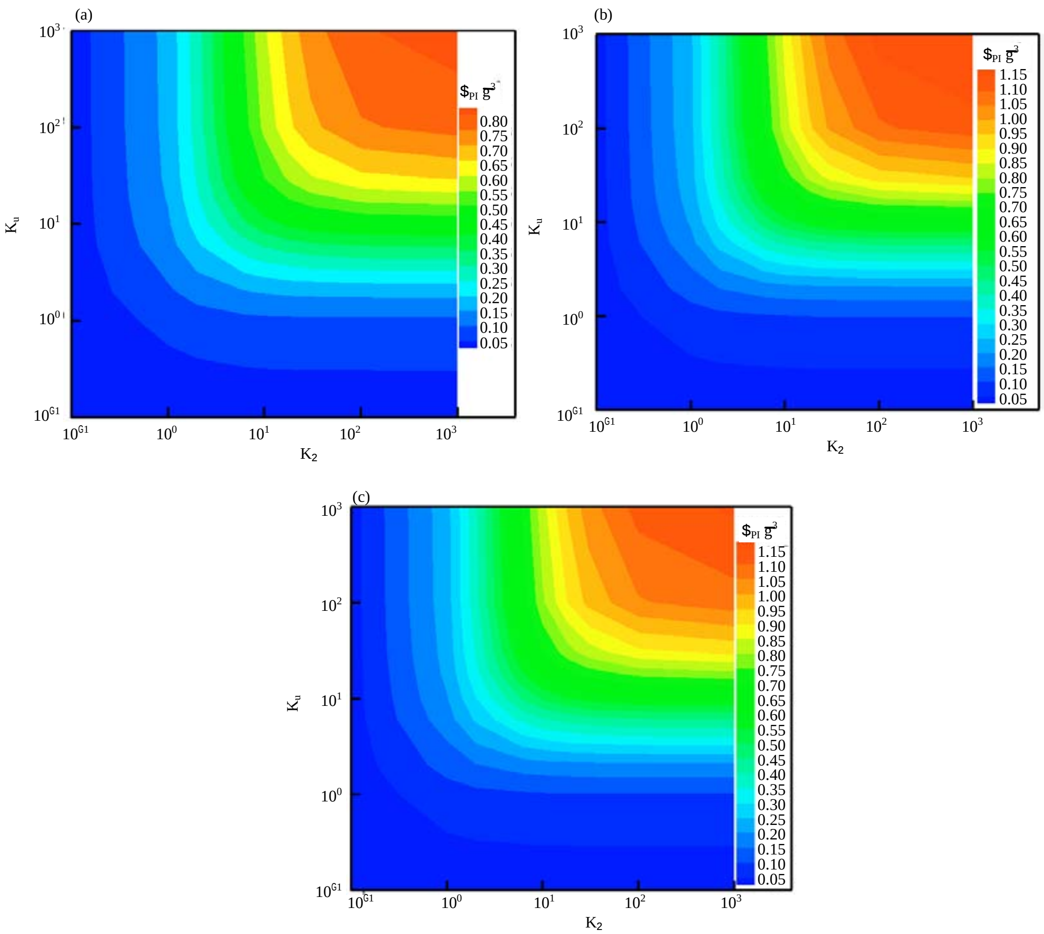

Fig. 2(a-c): Variation of $\beta_{PI}/\bar{g}^3$ of micro-beam for as a function of spring stiffness without considering vdW force and size effect (α = 0 and η = 0) obtained by; (a) Lumped model, (b) MAD and (c) Numerical method

It should be mentioned that the constitutive governing equation of cantilever single-layer conductive micro-actuator (Soroush *et al.*, 2010; Abadyan *et al.*, 2010; Moghimi Zand and Ahmadian, 2009) can be obtained from Eq. 12 by setting $\bar{g}=1$ and considering $K_u = K_\theta = 8$.

Figure 2 shows the variation of dimensionless pull-in voltage ($\beta_{PI}/\bar{g}^3$) of micro-beam for $\gamma\bar{g} = 0.65$ as a function of spring stiffness without considering vdW force (α = 0) and size effect (η = 0). As seen from this Figure, enhancing the spring stiffness leads to increase in pull-in voltage of micro-beam. Although, vdW force is usually neglected in MEMS simulation, size dependency may highly influence the pull-in performance of electromechanical systems even in micrometer ranges (Fleck *et al.*, 1994).

Figure 3 shows the size dependency of the pull-in voltage for supported micro-beams, without considering vdW force (α = 0). This Figure reveals that size effect (η = 0.5) increases pull-in voltage of the micro-actuator. Figure 2 and 3 show that MAD results are very close to numerical ones.

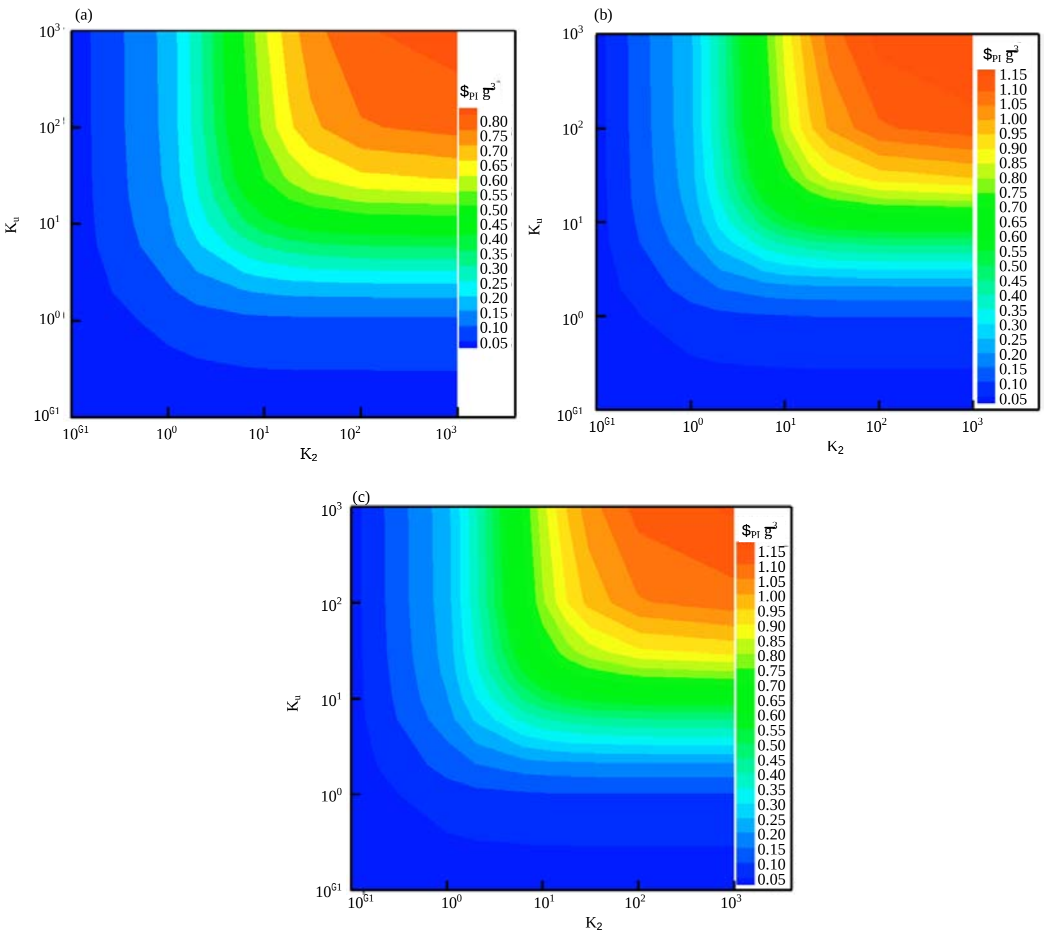

Fig. 3(a-c): Size dependency of the $\beta_{PI}/\bar{g}^3$ for micro-beams (α = 0) with η = 0.5 obtained by; (a) Lumped model, (b) MAD and (c) Numerical method

Single-layer nano-actuator: At nano-scale distances, both vdW attraction and size effect should be accounted in computing pull-in parameters of beam-type nano-actuators. Consider two applicable important cases of a single-layer conductive nano-actuator; One is ultra thin beam case, i.e., h<<g, where vdW force can be simplified in a form which approximately is proportional to the forth power of the separation. The other case is ultra small gap case, i.e., g<<h, where a classic form of vdW force (Batra *et al*., 2009) can be obtained which approximately is proportional to the inverse cube of the separation. For these cases, one can rewrite Eq. 8a as:

$$\frac{d^4\hat{w}}{dx^4} = \frac{\beta}{(1+\eta)(1-\hat{w}(x))^2} + \frac{\gamma\beta}{(1+\eta)(1-\hat{w}(x))} + \begin{cases} \dfrac{[3\alpha h/g]}{(1+\eta)(1-\hat{w}(x))^4} & \text{Thin beam}\,(h \ll g) \\ \dfrac{\alpha}{(1+\eta)(1-\hat{w}(x))^3} & \text{Smallgap}\,(g \ll h) \end{cases} \tag{13}$$

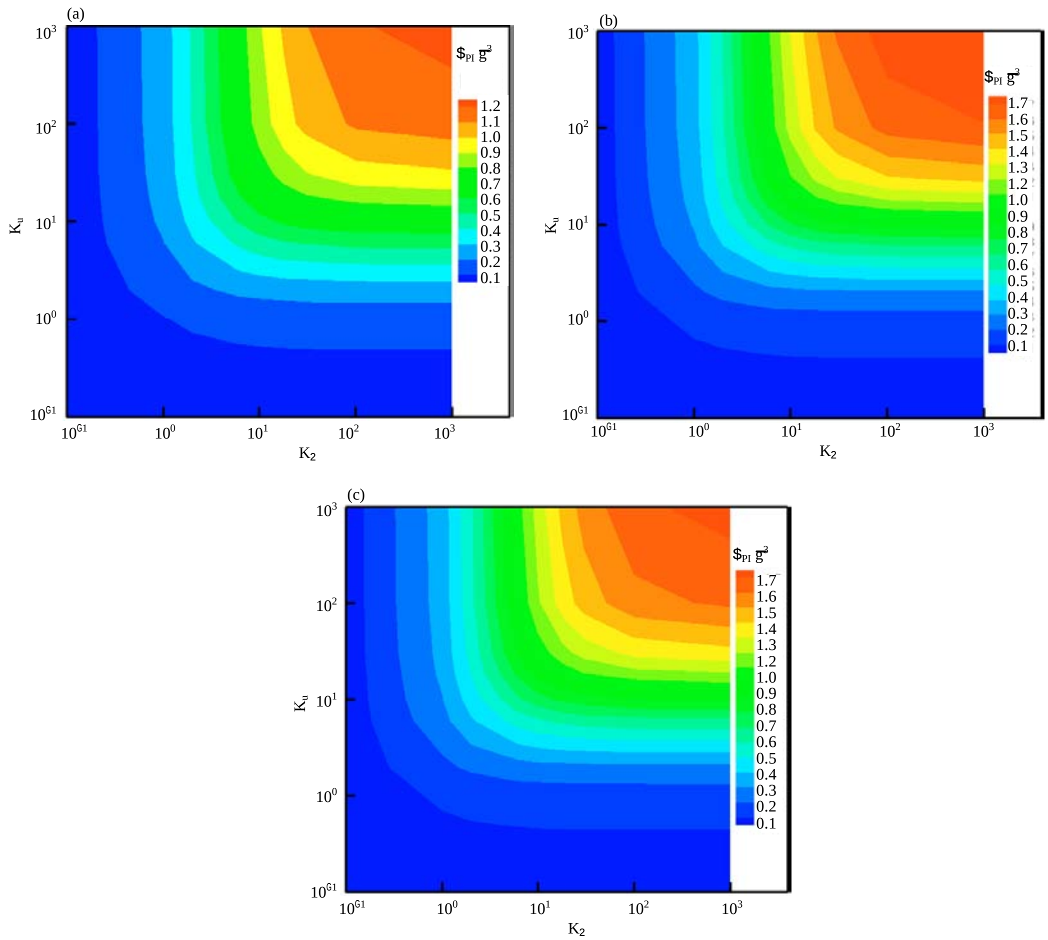

Fig. 4(a-c): The size dependency (η = 0.5) of the pull-in voltage of ultra-thin NEMS considering vdW force (3 αh/g = 0.5). Results obtained by: (a) Lumped model, (b) MAD and (c) Numerical method. Size effect increases pull-in voltage of the NEMS

Note that the common constitutive governing equation of cantilever single-layer nano-actuator (Soroush *et al.*, 2010) can be obtained from Eq. 13 by setting $\eta = 0$ and considering small gap case with $K_u = K_\theta = \infty$. Figure 4 and 5 depict the size dependency (η = 0.5) of the pull-in voltage of supported nano-actuator considering the effect of vdW force. Figure 4 corresponds to ultra thin actuators and Fig. 5 shows the results for ultra small separations. These Figures reveal that the size dependency of material characteristics increases pull-in voltage of the nano-actuator.

As seen from Fig. 2-5, the proposed modified couple stress-based model is capable to simulate the effect of size dependency of material characteristics, vdW attraction and non-classic B.C. on pull-in performance of multi-layer micro/nano-actuators. However, it is note worthy that the crucial assumptions used to derive the governing equation should be concerned to ensure reliable and accurate results in design electromechanical systems, especially in submicron scales where size effects and nonlinear behaviors appear. Interestingly, the analytical MAD solution is more reliable than lumped parameter model in calculating the pull-in voltage of the actuators. The proposed analytical model avoids time-consuming iterations and makes parametric studies possible in design processes.

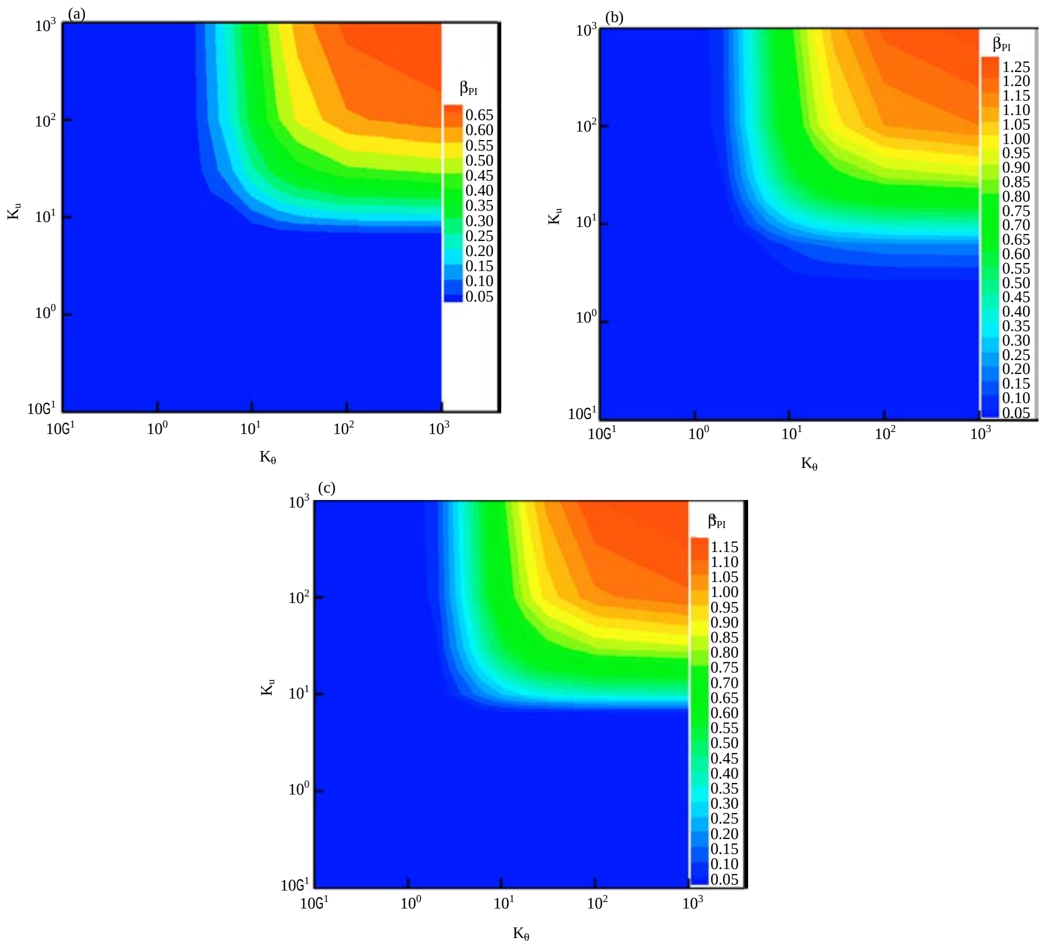

Fig. 5(a-c): The size dependency (η = 0.5) of the pull-in voltage of nano-actuator with ultra small separations considering vdW force (α = 0.5). Results obtained by: (a) Lumped model, (b) MAD and (c) Numerical method

CONCLUSIONS

The modified couple stress theory has been applied to model the effects of size dependency, vdW attraction and elastic B.Cs on the pull-in behavior of multilayer MEMS/NEMS. The following points can be concluded:

- By using modified couple stress theory, it is found that the dimensionless pull-in voltage of MEMS/NEMS increases linearly due to the size effect. This emphasizes the importance of size effect consideration in design and analysis of MEMS/NEMS
- The proposed model is capable of easy simulating the effect of elastic B.C using rotational and translational virtual springs. Results show that the instability voltage of MEMS/NEMS strongly depends on the spring coefficients
- The vdW attraction decreases the pull-in voltage of NEMS/MEMS. For ultra-thin single-layer model, vdW attraction approximately is proportional to the forth power of the separation while for ultra-small gap model, common cube dependency is obtained

- The analytical MAD solution is more accurate than the lumped parameter model and its difference with the numerical solution is within the excellent range for engineering applications

APPENDIX A

Consider a multi-layer beam where E (z) are Young's modulus function which may be defined as:

$$E(z)=\begin{cases} E_1 & z_0 \le z \le z_1 \\ \vdots & \vdots \\ E_n & z_{n-1} \le z \le z_n \end{cases} \tag{A.1}$$

In order to determine the position of the beam neutral axis z_c, equilibrium equation along X can be considered as (Lin and Zhao, 2005):

$$\int_{z_0}^{z_1} E_1 zbdz + \int_{z_1}^{z_2} E_2 zbdz + \ldots + \int_{z_{n-2}}^{z_{n-1}} E_{n-1} zbdz + \int_{z_{n-1}}^{z_n} E_n zbdz = 0 \tag{A.2}$$

where, $z_0 = 0\text{-}z_c$ and:

$$z_i = [\sum_{j=1}^{i} h_j] - z_c; 1 \le i \le n \tag{A.3}$$

Therefore (Rong *et al*., 2004):

$$z_c = \frac{1}{2}\frac{\sum_{i=1}^{n} E_i \left([\sum_{j=1}^{i} 2h_j] - h_i \right) h_i}{\sum_{i=1}^{n} E_i h_i} \tag{A.4}$$

APPENDIX B

Based on work by Lin and Zhao (2005), the lumped parameter model assume a uniform distribution for electrical and vdW force. Furthermore, this model only simulates the tip deflection of the beam and $\hat{\mathbf{w}}$ (X) is identical to $\hat{\mathbf{w}}_{tip}$ for lumped model. Therefore, the governing equation of the lumped parameter model is:

$$K_{eff}\, w_{tip} = f_{elec} + f_{vdW} \tag{B.1}$$

where, K_{eff} is the effective stiffness of the actuator. As shown in Fig. B.1, K_{eff} can be obtained from superposition of the translational, rotational and bending stiffness of the structure (i.e., K_T, K_R and K_B, respectively) according to the following relation:

$$K_{eff}^{-1} = K_T^{-1} + K_R^{-1} + K_B^{-1} = \frac{1}{K_u} + \frac{L^2}{2K_\theta} + \frac{L^3}{8\left(\frac{b}{3}\sum_{i=1}^{n} E_i \left(z_i^3 - z_{i-1}^3 \right) + b\sum_{i=1}^{n} \mu_i l_i^2 (z_i - z_{i-1}) \right)} \tag{B.2}$$

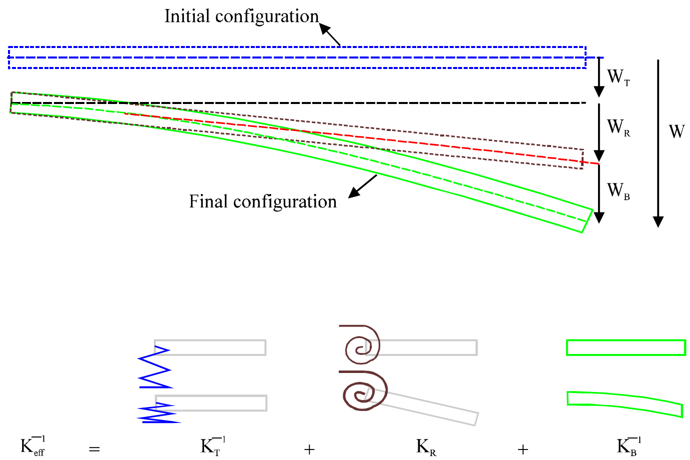

Fig. B.1: Derivation of equivalent stiffness coefficient for a lumped parameter model

Substituting Eq. B.2 in B.1 and using the same notation defined in the text, the governing equation of the lumped parameter model is:

$$\left(\frac{8}{1+4\left(\frac{2}{\bar{K}_u}+\frac{1}{\bar{K}_\theta}\right)}\right)\hat{w}_{tip}=\frac{\beta}{(1+\eta)\left(\bar{g}-\hat{w}_{tip}\right)^2}+\frac{\gamma\beta}{(1+\eta)\left(\bar{g}-\hat{w}_{tip}\right)}+\frac{1}{(1+\eta)}\sum_{i=1}^{n}\left[\frac{\alpha_i}{\left(\hat{g}_{i-1}-\hat{w}_{tip}\right)^3}-\frac{\alpha_i}{\left(\hat{g}_i-\hat{w}_{tip}\right)^3}\right] \tag{B.3}$$

In order to obtain pull-in voltage of lumped parameter model, Eq. B.3 may be rewritten as relation (10):

APPENDIX C

Consider a forth order nonlinear boundary value problem of:

$$L^{(4)}\,[y\,(x)] = N\,(x,\,y),\ 0\le x\le L_b \tag{C.1}$$

$$y^{(0)} = C_0,\ y'\,(0) = C_1 \tag{C.2}$$

where, the differential operator L(4) and the corresponding inverse L(-4) are defined as:

$$L^{(4)}=\frac{d^4}{dx^4}(\ ) \tag{C.3.1}$$

$$L^{(-4)}=\int_0^x\int_0^x\int_0^x\int_0^x(\ )dx\,dx\,dx\,dx \tag{C.3.2}$$

In order to employ MAD method, nonlinear function N (x,y) is represented as a series of Adomian polynomials (Kuang and Chen, 2005):

$$N(x,y)=\sum_{n=0}^{\infty}N_n(x) \tag{C.4}$$

where, series N_n which approximates the nonlinear function N is determined by the following convenient equations (Kuang and Chen, 2005):

$$N_n=\sum_{v=1}^{n}C(v,n)A_v(y_0),(n>0) \tag{C.5}$$

$$C(v,n)=\sum_{p_i}\prod_{i=1}^{v}\frac{1}{k!}y_{p_i}^{k_i},\ (\sum_{i=1}^{v}k_ip_i=n,\ 0\le i\le n,\ 1\le p_i\le n-v+1) \tag{C.6}$$

$$A_v(y_0)=\frac{d^v}{dy_0^v}[N(y_0)] \tag{C. 7}$$

where, k_i is the number of repetition in y_{pi} and the values of pi are selected from the above range by combination. Expanding (C.5-C.7) yields:

$$\begin{aligned}
&N_0=A_0(y_0)\\
&N_1=C(1,1)A_1(y_0)=y_1A_1(y_0)\\
&N_2=C(1,2)A_1(y_0)+C(2,2)A_2(y_0)=y_2A_1(y_0)+\frac{1}{2!}y_1^2A_2(y_0)\\
&N_3=C(1,3)A_1(y_0)+C(2,3)A_2(y_0)+C(3,3)A_3(y_0)=y_3A_1(y_0)+y_1y_2A_2(y_0)+\frac{1}{3!}y_1^3A_3(y_0)\\
&\ldots\ldots
\end{aligned} \tag{C.8}$$

Hence, the dependent variable in Eq. C.1 and C.2 can be written as:

$$y(x)=\sum_{n=0}^{\infty}y_n(x)=C_0+C_1x+\frac{1}{2!}C_2x^2+\frac{1}{3!}C_3x^3+L^{(-4)}\left[\sum_{n=0}^{\infty}N_n(x)\right] \tag{C.9}$$

where, the constants C_2 and C_3 can be determined by solving the resulted algebraic equation from B.C. at $x = L_b$. Referring to MAD method, the recursive relations of Eq. C.9 can be provided as follows by Kuang and Chen (2005):

$$\begin{aligned}
&y_0=C_0,\\
&y_1=C_1x+\frac{1}{2!}C_2x^2+\frac{1}{3!}C_3x^3+L^{(-4)}[N_0]\\
&y_{n+1}=L^{(-4)}[N_n(x)]
\end{aligned} \tag{C.10}$$

Using the above recursive relations, the solution of Eq. (C.1) is easily obtained.

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