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A Short Note on Intuitionistic Fuzzy Ternary Subpolygroups

Bijan Davvaz
Department of Mathematics, Yazd University, Yazd, Iran

ABSTRACT

After the introduction of fuzzy sets by Zadeh, several researchers were conducted on the generalization of fuzzy sets. The notion of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of the notion of fuzzy sets. An intuitionistic fuzzy set for a given universal set is defined by a degree of membership function and a degree of non-membership function. The notion of ternary polygroup is a generalization of the notion of polygroup in the sense of Comer. In this research, we study the concept of intuitionistic fuzzy ternary subpolygroups of a ternary polygroup.

Key words: Hypergroup, polygroup, ternary polygroup, intuitionistic fuzzy set

INTRODUCTION

The theory of algebraic hyperstructures which is a generalization of the concept of algebraic structures first was introduced by Marty (1934) and had been studied in the following decades and nowadays by many mathematicians and many papers concerning various hyperstructures have appeared in the literature. The basic definitions of the object can be found (Corsini, 1993; Corsini and Leoreanu, 2003). A hyperstructure is a non-empty set H together with a map $\cdot: H \times H \to P^*(H)$ called hyperoperation, where P*(H) denotes the set of all the non-empty subsets of H. A hyperstructure (H,·) is called a hypergroup if the following axioms hold:

- $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ for all x, y, z∈H
- $a \cdot H = H \cdot a = H$ for all a∈H

If x∈H and A, B are non-empty subsets of H, then by A.B, A.x and x.B we mean:

$$A \cdot B = \bigcup_{\substack{a\in A \\ b\in B}} a \cdot b,\ A \cdot x = A \cdot \{x\} \text{ and } x \cdot B = \{x\} \cdot B$$

The concept of fuzzy sets was introduced by Zadeh (1965). Let X be a set. A fuzzy subset A of X is characterized by a membership function $\mu_A: X \to [0, 1]$, which associates with each point x∈X its grade or degree of membership $\mu_A(x) \in [0, 1]$. Let A and B be two fuzzy subsets of X. Then:

- A = B if and only if $\mu_A(x) = \mu_B(x)$, for all x∈X
- A⊆B if and only if $\mu_A(x) \le \mu_B(x)$, for all x∈X
- C = A∪B if and only if $\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}$, for all x∈X
- D = A∩B if and only if $\mu_D(x) = \min\{\mu_A(x), \mu_B(x)\}$ for all x∈X
- The complement of A, denoted by A^c is defined by $\mu_{A^c}(x) = 1-\mu_A(x)$. for all x∈X

Fuzzy set theory and its applications in several branches of science are growing day by day. These applications can be found in various fields such as computer science, artificial intelligence, operation research, management science, control engineering, expert systems and many others, for example (Ali, 2011; Ayanzadeh *et al.*, 2012; Dehini *et al.*, 2012; Fatemi, 2011; Saad *et al.*, 2007; Yufeng *et al.*, 2011).

Rosenfeld (1971) applied this concept to the theory of groups and studied fuzzy subgroups of a group (Ersoy *et al.*, 2002; Fathi and Salleh, 2009; Massadehss, 2011). Davvaz (1999) applied fuzzy sets to the theory of algebraic hyperstructures and studied their fundamental properties. Further investigations are contained in many papers, for example see the list of references.

Definition 1: Let $(H,\cdot)$ be a hypergroup and let μ a fuzzy subset of H. Then, μ is said to be a fuzzy subhypergroup of H if the following axioms hold:

- $\min\{\mu(x),\mu(y)\} \le \inf_{z\in x\cdot y}\{\mu(z)\}$ for all $x, y\in H$
- for all $x, a\in H$ there exists $y\in H$ such that $x \in a\cdot y$ and $\min\{\mu(a), \mu(x)\}\le\mu(y)$
- for all $x, a\in H$ there exists $z\in H$ such that $x \in z\cdot a$ and $\min\{\mu(a), \mu(x)\}\le\mu(z)$

POLYGROUPS AND TERNARY POLYGROUPS

Application of hypergroups have mainly appeared in special subclasses. For example, polygroups which are certain subclasses of hypergroups are studied by Comer (1984) and are used to study color algebra. Quasi-canonical hypergroups (called polygroups by Comer) were introduced by Bonansinga and Corsini (1982), as a generalization of canonical hypergroups introduced by Mittas (1972). Some algebraic and combinatorial properties were developed by Comer. We recall the following definition from Comer (1984). A polygroup is a multi-valued system $\langle P,*,e,^{-1}\rangle$ where $^{-1}:P\rightarrow P$ and $*$ is a hyperoperation from $P\times P$ into the family of non-empty subsets of P such that the following axioms hold:

- $(x*y)*z = x*(y*z)$ for all $x, y, z\in P$
- $e*x = x*e = x$
- $x\in y*z$ implies $y\in x*z^{-1}$ and $z\in y^{-1}*x$

Zahedi *et al.* (1995) defined the concept of fuzzy subpolygroups of a polygroup which is a generalization of the concept of Rosenfeld's fuzzy subgroups and special case of Davvaz's for fuzzy subhypergroups. Let $\langle P,*,e,^{-1}\rangle$ be a polygroup and let μ a fuzzy subset of P. Then μ is definition said to be a fuzzy subpolygroup of P if the following axioms hold:

- $\min\{\mu(x), \mu(y)\}\le\mu(z)$ for all $x, y\in P$ and for all $z\in x*y$
- $\mu(x)\le\mu(x^{-1})$ for all $x\in P$

The concept of n-ary hypergroup is defined by Davvaz and Vougiouklis (2006), which is a generalization of the concept of hypergroup in the sense of Marty and a generalization of n-ary group, too. Davvaz and Corsini (2007) introduced the notion of a fuzzy n-ary subhypergroup of an n-ary hypergroup. Then this concept studied (Davvaz *et al.*, 2009; Davvaz and Leoreanu-Fotea, 2010a, b; Ghadiri and Waphare, 2009; Kazanci *et al.*, 2010, 2011; Zhan *et al.*, 2010). A ternary hypergroup is a particular case of an n-ary hypergroup for n = 3. Davvaz and Leoreanu-Fotea

(2010b) studied the ternary hypergroups associated with a binary relations. Davvaz *et al.* (2011) provided examples of ternary hyperstructures associated with chain reactions in chemistry (also, see, Davvaz (2009)).

Let H be a non-empty set and $f: H\times H\times H \rightarrow P^*(H)$. Then f is called a ternary hyperoperation on H and the pair (H, f) is called a ternary hypergroupoid. If A, B, C are non-empty subsets of H, then we define:

$$f(A, B, C) = \bigcup_{a\in A,\, b\in B,\, c\in C} f(a, b, c)$$

The ternary hypergroupoid (H, f) is called a ternary semihypergroup if for every $a_1,...a_5 \in H$, we have:

$$f(f(a_1, a_2, a_3), a_4, a_5) = f(a_1, f(a_2, a_3, a_4), a_5) = f(a_1, a_2, f(a_3, a_4, a_5))$$

A ternary semihypergroup (H, f) is called a ternary hypergroup if for all $a, b, c \in H$ there exist $x, y, z \in H$ such that:

$$c \in f(x, a, b) \cap f(a, y, b) \cap f(a, b, z)$$

Notice that a ternary semigroup (S, f) is said to be a ternary group if it satisfies the following property that for all $a, b, c \in S$, there exist unique $x, y, z \in S$ such that:

$$c = f(x, a, b),\ c = f(a, y, b),\ c = f(a, b, z)$$

A ternary polygroup is a multi-valued system $\langle P, f, e, ^{-1}\rangle$ where $e \in P$, $^{-1}: P \rightarrow P$ is a unitary operation and f is a ternary hyperoperation from $P\times P\times P$ into the family of non-empty subsets of P such that the following axioms hold:

$$f\,(f\,(\alpha_1, \alpha_2, \alpha_3), \alpha_4, \alpha_5) = f\,(\alpha_1, \alpha_2, \alpha_3, \alpha_4), \alpha_5) = f\,(\alpha_1, \alpha_2, f\,(\alpha_3, \alpha_4, \alpha_5)) \quad (1)$$

for every $\alpha_1,..., \alpha_5 \in P$, e is a unique element such that $f\,(x, e, e) = f\,(e, x, e) = x$ for every $x \in P$ and $e^{-1} = e$, $z \in f\,(x_1, x_2, x_3)$ implies $x_1 \in f\,(z, x_2^{-1}, x_3^{-1})$, $x_2 \in f\,(x_1^{-1}, z, x_3^{-1})$ and $x_3 \in f\,(x_1^{-1}, x_2^{-1}, z)$.

INTUITIONISTIC FUZZY SETS

We recollect some relevant basic preliminaries and in particular, the study of Atanassov (1986). Let X be a fixed set. An intuitionistic fuzzy set A in X is an object having the form:

$$A = \{<x, \mu_A(x), \lambda_A(x)> \mid x \in X\}$$

where, the functions $\mu_A: X \rightarrow [0,1]$ and $\lambda_A: X \rightarrow [0,1]$ are the degree of membership and the degree of non-membership of the element $x \in X$ to the set A, respectively; moreover, $0 \le \mu_A(x)+\lambda_A(x) \le 1$ must hold. Note that a Zadeh fuzzy set, written down as an intuitionistic one, is of the form:

$$A = \{<x, \mu_A(x), 1-\mu_A(x)> \mid x \in X\}$$

Let X be a non-empty set and let $A = \{<x, \mu_A(x), \lambda_A(x)> \mid x \in X\}$ and $B = \{<x, \mu_B(x), \lambda_B(x)> \mid x \in X\}$ be two intuitionistic fuzzy sets. Then:

- $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$ for all $x \in X$
- $A = B$ iff $A \subseteq B$ and $B \subseteq A$
- $A^c = \{<x, \lambda_A(x), \mu_A(x)> \mid x \in X\}$
- $A \cap B = \{<x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\}> \mid x \in X\}$
- $A \cup B = \{<x \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\}> \mid x \in X\}$
- $\Box A = \{<x, \mu_A(x), 1-\mu_A(x)> \mid x \in X\}$
- $\Diamond A = \{<x, 1-\lambda_A(x), \lambda_A(x)> \mid x \in X\}$

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ or $A = (\mu, \lambda)$ for intuitionistic fuzzy set $A = \{<x, \mu_A(x), \lambda_A(x)> \mid x \in X\}$.

The concept of intuitionistic fuzzy subgroup of a group is introduced by Biswas (1989). Let G be an ordinary group. An intuitionistic fuzzy $A = (\mu_A, \lambda_A)$ set in G is called an intuitionistic fuzzy subgroup of G if:

- $\min\{\mu_A(x), \mu_A(y)\} \leq \mu_A(xy)$ for all $x, y \in G$
- $\mu_A(x) \leq \mu_A(x^{-1})$ for all $x \in G$
- $\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\}$ for all $x, y \in G$
- $\lambda_A(x^{-1}) \leq \lambda_A(x)$ for all $x \in G$

INTUITIONISTIC FUZZY TERNARY SUBPOLYGROUPS

Definition 2: Let $\langle P, f, e^{-1} \rangle$ be a ternary polygroup and $A = (\mu, \lambda)$ be an intuitionistic fuzzy subset of P. Then, μ is said to be an intuitionistic fuzzy ternary subpolygroup of P if the following axioms hold:

- $\min\{\mu(x), \mu(y), \mu(z)\} \leq \inf_{z \in f(x,y,z)}\{\mu(z)\}$ for all $x, y, z \in P$
- $\mu(x) \leq \mu(x^{-1})$ for all $x \in P$
- $\sup_{\alpha \in f(x,y,z)}\{\mu(z)\} \leq \max\{\mu(x), \mu(y), \mu(z)\}$ for all $x, y, z \in P$
- $\mu(x^{-1}) \leq \mu(x)$ for all $x \in P$

For any fuzzy set μ of H and any $t \in [0, 1]$ we define two sets:

$$U(\mu; t) = \{x \in H \mid \mu(x) \geq t\} \text{ and } L(\mu; t) = \{x \in H \mid \mu(x) \leq t\}$$

which are called an upper and lower t-level cut of μ and can be used to the characterization of μ.

Theorem 1: Let $\langle P, f, e^{-1} \rangle$ be a ternary polygroup and $A = (\mu, \lambda)$ be an intuitionistic fuzzy subset of P. Then, $U(\mu; t)$ and $L(\lambda; t)$ are subpolygroups of P for every $t \in Im(\mu_A) \cap Im(\lambda_A)$.

Proof: Suppose that $A = (\mu, \lambda)$ is an intuitionistic fuzzy ternary subpolygroup of P. For every x, y, $\in U(\mu; t)$ we have $\min\{\mu(u), \mu(y), \mu(z)\} \geq t$ and so $\inf_{\alpha \in f(z, y, z)}\{\mu(\alpha)\} \geq t$. Thus, for every x, y z, $\in U(\mu; t)$ we have $\mu(\alpha) \geq t$. Therefore, $f(x, y, z) \subseteq U(\mu; t)$. Now, if $x \in U(\mu; t)$ then $t \geq \mu(x)$ Since $\mu(x) \leq \mu(x^{-1})$ we conclude that which implies that $x^{-1} \in U(\mu; t)$.

Also, for every x, y, z ϵL (μ; t) we have max {λ(x), λ(y), λ(z)} and so:

$$\sup_{\alpha\in(x,\, y,\, z)} \{\lambda(\alpha)\} \leq t$$

Thus, for every αϵf(x, y, z) we have $\lambda(\alpha)\leq t$. Therefore, f (x, y, z)⊆L (μ; t). Now, if xϵL (μ; t) then λ (x)≤t. Since $\lambda(x^{-1})\leq\lambda(x)$ we conclude that $\lambda(x^{-1})\leq t$ which implies that x^{-1} ϵL (μ; t).

Theorem 2: Let ⟨P, f, e, $^{-1}$⟩ be a ternary polygroup and A = (μ, λ) be an intuitionistic fuzzy subset of P such that the non-empty sets U (μ; t) and L (λ; t) are ternary subpolygroups of P for all tϵ[0, 1]. Then, A = (μ, λ) is an intuitionistic fuzzy ternary subpolygroup of P.

Proof: Assume that for every 0≤t≤1, U (μ; t) (≠ϕ) is a ternary subpolygroup of P. For every x, y, zϵP, we put t_0 = min {μ (x), μ (y), μ (z)}. Then x, y, zϵU (μ; t) and so f (x, y, z)⊆U(μ; t). Therefore, for every αϵf (x, y, z) we have $\mu(\alpha)\geq t_0$ implying that:

$$\min\{\mu(x), \mu(y), \mu(z)\} \leq \inf_{\alpha\in(x,\, y,\, z)} \{\mu(\alpha)\}$$

and in this way the first condition of Definition 2 is verified. In order to verify the second condition, let xϵP. We put $t_1 = \mu(x)\}$ Since U(μ; t) is a ternary subpolygroup, x^{-1} ϵU (μ; t_1), which implies that μ(x), $\leq\mu(x^{-1})$

Now, suppose that for every 0≤t≤1, L (λ; t) (≠∅) is a ternary subpolygroup of P. For every x, y, z, ϵP, we put t0 = max {λ(x), λ(y), λ(z)}. Then x, y, z, ϵ L (μ; t_0) and so f (x, y, z)⊆L (μ; t_0). Therefore, for every αϵf (, y, z) we have $\lambda(\alpha)\leq t_0$ implying that:

$$\sup_{\alpha\in(x,\, y,\, z)} \{\lambda(\alpha)\} \leq \max\{\lambda(x), \lambda(y), \lambda(z)\}$$

and in this way the third condition of Definition 2 is verified. In order to verify the last condition, let xϵP. We put $t_1 = \lambda(x)\}$ Since L (λ; t) is a ternary subpolygroup, x^{-1} ϵL(λ; t_1), which implies that $\lambda(x^{-1})\leq\lambda(x)$.

Corollary 1: Let χ_K be the characteristic function of a ternary subpoly K of P. Then, K = (χ_K, $\chi^c{}_k$) is an intuitionistic fuzzy ternary subpolygroup of P.

Corollary 2: Let ⟨P, f, e, $^{-1}$⟩ be a ternary polygroup. Then A = (μ, λ) is an intuitionistic fuzzy ternary subpolygroup of P if and only if □ A and ◊A are intuitionistic fuzzy ternary subhypergroup of P.

Proof: Suppose that A = (μ, λ) is an intuitionistic fuzzy ternary polygroup of P. For every x, y, z in P, we have:

- min {μ(x), μ(y), μ(z)}≤ $\inf_{\alpha\epsilon f\,(x,\, y,\, z)}$ {μ (α)}, or
- min {1-μ^c (x), 1-μ^c (y), 1-μ^c (z)}≤ $\sup_{\alpha\epsilon f\,(x,\, y,\, z)}$ {1-μ^c (α)}, or
- min {1-μ^c (x), 1-μ^c (y), μ^c (z)}≤ $\sup_{\alpha\epsilon f\,(x,\, y,\, z)}$ {μ^c (α)}, or
- $\sup_{\alpha\epsilon f\,(x,\, y,\, z)}$ {μ^c (x)≤ 1-min (1-μ^c(x), 1-μ^c (y), 1-μ^c (z)}, or
- $\sup_{\alpha\epsilon f\,(x,\, y,\, z)}$ {μ^c (α)≤ max (μ^c (x), μ^c (y), μ^c (z)}.

Since μ is a fuzzy ternary subpolygroup of P, so for every x∈P, $\mu(x)\alpha\mu(x^{-1})$ or $1-\mu(x^{-1})\le 1-\mu(x)$ which implies that. The converse also can be proved similarly.

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