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New Application of Direct Algebraic Method to Eckhaus Equation

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ABSTRACT

In this present study, we applied new applications of direct algebraic method to Eckhaus equation, the balance numbers of which are not positive integers. Then new types of complex solutions are obtained to the Eckhaus equation.

Key words: Direct algebraic method, Eckhaus equation, complex solutions

INTRODUCTION

Seeking the complex and exact solutions of nonlinear partial differential equations plays an important role in nonlinear problems. When we want to understand the physical mechanism of phenomena in nature, described by nonlinear evolution equations, exact solutions for the nonlinear evolution equations have to be explored. Recently many new approaches to obtain the exact solutions of nonlinear differential equations have been proposed. When we want to understand the physical mechanism of phenomena in nature, described by nonlinear evolution equations, exact solutions for the nonlinear evolution equations have to be explored. Thus, the methods for deriving exact solutions for the governing equations have to be developed. Recently, many powerful methods have been established and improved. Among these methods, we cite the homogeneous balance method (Wang, 1995; Wang *et al*., 1996), the tanh-function method (Parkes and Duffy, 1996), the extended tanh-function method (Fan, 2000), the Jacobi elliptic function expansion method (Liu *et al*., 2001; Fu *et al*., 2003), the auxiliary equation method (Sirendaoreji, 2004) and so on (Salekdeh *et al*., 2012; Pramada *et al*., 2012; Rokrok *et al*., 2012; Iwueze and Ohakwe, 2011; Ahmad *et al*., 2011; Zainodin *et al*., 2011). Recently, the direct algebraic method and symbolic computation have been suggested to obtain the exact complex solutions of nonlinear partial differential equations (Kufre *et al*., 2011; Jabbari and Solookinejad, 2011; Kazemi *et al*., 2011; Maina *et al*., 2011; Ferrari *et al*., 2000; Zhang, 2009).

DESCRIPTION OF DIRECT ALGEBRAIC METHOD

For a given partial differential equation:

$$G\,(u,\, u_x,\, u_t,\, u_{xx},\, u_{tt},\ldots) \tag{1}$$

Our method mainly consists of four steps:

Step 1: We seek complex solutions of Eq. 1 as the following form:

$$u = u(\xi),\ \ \xi = ik\,(x-ct) \tag{2}$$

where, k and c are real constants. Under the transformation (2), Eq. 1 becomes an ordinary differential equation:

$$N\,(u,\ ikcu',\ -ikcu',\ -k^2u'',...) \tag{3}$$

Where:

$$u' = \frac{du}{d\xi}$$

Step 2: We assume that the solution of Eq. 3 is of the form:

$$u\,(\xi) = \sum_{i=0}^{n} a_i F^i\,(\xi) \tag{4}$$

where, a_i (i = 1, 2,..., n) are real constants to be determined later. F(ξ) expresses the solution of the auxiliary ordinary differential equation:

$$F'(\xi) = b+F^2\,(\xi) \tag{5}$$

Eqution 5 admits the following solutions:

$$\begin{aligned} F(\xi) &= \begin{cases} -\sqrt{-b}\tanh(\sqrt{-b}\xi), & b \prec 0 \\ -\sqrt{-b}\coth(\sqrt{-b}\xi), & b \prec 0 \end{cases} \\ F(\xi) &= \begin{cases} \sqrt{b}\tan(\sqrt{b}\xi), & b \succ 0 \\ -\sqrt{b}\cot(\sqrt{b}\xi), & b \succ 0 \end{cases} \\ F(\xi) &= -\frac{1}{\xi}, \qquad b = 0 \end{aligned} \tag{6}$$

Integer n in (4) can be determined by considering homogeneous balance [3] between the nonlinear terms and the highest derivatives of u(ξ) in Eq. 3.

Step 3: Substituting (4) into (3) with (5), then the left hand side of Eq. 3 is converted into a polynomial in F(ξ), equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, k, c.

Step 4: Solving the algebraic equations obtained in step 3 and substituting the results into (4), then we obtain the exact traveling wave solutions for Eq. 1.

APPLICATION TO ECKHAUS EQUATION

The Eckhaus equation reads:

$$i\Phi_t+\Phi_{xx}+2(|\Phi|^2)_x\Phi+|\Phi|\Phi = 0, \qquad \Phi:\mathbb{R}\to\mathbb{C} \tag{7}$$

We may choose the following complex travelling wave transformation:

$$\Phi = u(\xi)\,e^{i(\alpha x+\beta t)}, \qquad \xi = ik\,(x-2\alpha t) \tag{8}$$

where, α, β are constants to be determined later. Using the complex traveling wave solutions (8) we have the nonlinear ordinary differential equation:

$$(i\beta-\alpha^2)\,u-2k\alpha(i+1)\,u'-k^2u''+4iku'u^2+u^5=0 \tag{9}$$

Considering the homogeneous balance between u^5 and u" in Eq. 9, we required that 5m = m+2⇒m = 1/2. It should be noticed that m is not a positive integer. However, we may still choose the solution of Eq. 7 in the form:

$$u = AF^{\frac{1}{2}} \tag{10}$$

So:

$$u' = \frac{1}{2}A\left[bF^{-\frac{1}{2}} + F^{\frac{3}{2}}\right] \tag{11}$$

$$u'' = \frac{1}{4}A\left[-b^2F^{-\frac{3}{2}} + 2bF^{\frac{1}{2}} + 3F^{\frac{5}{2}}\right] \tag{12}$$

$$\phi^2\phi' = \frac{1}{2}A^3\left[bF^{\frac{1}{2}} + F^{\frac{5}{2}}\right] \tag{13}$$

Substituting Eq. 10-13 into Eq. 9, we obtain:

$$A^4 + 2ikA^2 - \frac{3}{4}K^2 = 0$$

$$2ikA^2b + i\beta - \alpha^2 - \frac{1}{2}K^2b = 0$$

By solving equation above we obtain:

$$\text{Case1}:\quad A_1 = \frac{\sqrt{-k}}{2}(i+1), \qquad \alpha_1 = \pm\sqrt{i\beta - \frac{1}{2}kb^2 + k^2b} \tag{14}$$

$$\text{Case2}:\quad A_2 = \frac{\sqrt{-3k}}{2}(i+1), \qquad \alpha_2 = \pm\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2b} \tag{15}$$

From Eq. 6, 10, 14 and 15, we obtain the complex travelling wave solutions of Eq. 7 as follows:

Case 1: For:

$$A_1 = \frac{\sqrt{-k}}{2}(i+1), \qquad \alpha_1 = \pm\sqrt{i\beta - \frac{1}{2}kb^2 + k^2b}$$

we obtain:

$$u_1 = \frac{\sqrt{-k}}{2}(i+1)\left[-\sqrt{-b}\tanh(\sqrt{-b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bt}))\right]^{\frac{1}{2}}$$

where, b<0 and k is an arbitrary real constant.

Hence:

$$\Phi_1 = \frac{\sqrt{-k}}{2}(i+1)\left[-\sqrt{-b}\tanh(\sqrt{-b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bt}))\right]^{\frac{1}{2}} e^{i(\pm\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bx} + \beta t)}$$

And:

$$u_2 = \frac{\sqrt{-k}}{2}(i+1)\left[-\sqrt{-b}\coth(\sqrt{-b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bt}))\right]^{\frac{1}{2}}$$

$$\Phi_2 = \frac{\sqrt{-k}}{2}(i+1)\left[-\sqrt{-b}\coth(\sqrt{-b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bt}))\right]^{\frac{1}{2}} e^{i(\pm\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bx} + \beta t)}$$

where, b<0 and k is an arbitrary real constant:

$$u_3 = \frac{\sqrt{-k}}{2}(i+1)\left[\sqrt{b}\tan(\sqrt{b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bt}))\right]^{\frac{1}{2}}$$

$$\Phi_3 = \frac{\sqrt{-k}}{2}(i+1)\left[\sqrt{b}\tan(\sqrt{b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bt}))\right]^{\frac{1}{2}} e^{i(\pm\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bx} + \beta t)}$$

where, b>0 and k is an arbitrary real constant:

$$u_4 = \frac{\sqrt{-k}}{2}(i+1)\left[-\sqrt{b}\cot(\sqrt{b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bt}))\right]^{\frac{1}{2}}$$

$$\Phi_4 = \frac{\sqrt{-k}}{2}(i+1)\left[-\sqrt{b}\cot(\sqrt{b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bt}))\right]^{\frac{1}{2}} e^{i(\pm\sqrt{i\beta - \frac{1}{2}kb^2 + k^2bx} + \beta t)}$$

where, b>0 and k is an arbitrary real constant:

$$u_5 = -\frac{\sqrt{-k}(i+1)}{2(x \mp 2\sqrt{i\beta}t)}$$

For b = 0.

Case 2: For:

$$A_2 = \frac{\sqrt{-3k}}{2}(i+1), \qquad \alpha_2 = \pm\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2b}$$

we obtain:

$$u_1 = \frac{\sqrt{-3k}}{2}(i+1)\left[-\sqrt{-b}\tanh(\sqrt{-b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2bt}))\right]^{\frac{1}{2}}$$

where, b<0 and k is an arbitrary real constant.

Hence:

$$\Phi_1 = \frac{\sqrt{-3k}}{2}(i+1)\left[-\sqrt{-b}\tanh(\sqrt{-b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2bt}))\right]^{\frac{1}{2}} e^{i(\pm\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2b}x + \beta t)}$$

And:

$$u_2 = \frac{\sqrt{-3k}}{2}(i+1)\left[-\sqrt{-b}\coth(\sqrt{-b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2bt}))\right]^{\frac{1}{2}}$$

$$\Phi_2 = \frac{\sqrt{-3k}}{2}(i+1)\left[-\sqrt{-b}\coth(\sqrt{-b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2bt}))\right]^{\frac{1}{2}} e^{i(\pm\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2b}x + \beta t)}$$

where, b<0 and k is an arbitrary real constant:

$$u_3 = \frac{\sqrt{-3k}}{2}(i+1)\left[\sqrt{b}\tan(\sqrt{b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2bt}))\right]^{\frac{1}{2}}$$

$$\Phi_3 = \frac{\sqrt{-3k}}{2}(i+1)\left[\sqrt{b}\tan(\sqrt{b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2bt}))\right]^{\frac{1}{2}} e^{i(\pm\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2b}x + \beta t)}$$

where, b>0 and k is an arbitrary real constant:

$$u_4 = \frac{\sqrt{-3k}}{2}(i+1)\left[-\sqrt{b}\cot(\sqrt{b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2bt}))\right]^{\frac{1}{2}}$$

$$\Phi_4 = \frac{\sqrt{-3k}}{2}(i+1)\left[-\sqrt{b}\cot(\sqrt{b}ik(x \mp 2\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2bt}))\right]^{\frac{1}{2}} e^{i(\pm\sqrt{i\beta - \frac{1}{2}kb^2 + 3k^2b}x + \beta t)}$$

where, b>0 and k is an arbitrary real constant:

$$u_5 = -\frac{\sqrt{-3k}(i+1)}{2(x \mp \sqrt{2\beta}(1+i)t)}$$

For b = 0.

CONCLUSION

The application of direct algebraic method was still limited to those equations the balance numbers of which are positive integers. In this study, we explore a new application of the direct algebraic method and obtain new types of complex wave solutions to the Eckhaus equation.

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