Trends in

Applied Sciences Research

ISSN 1819-3579

Academic
Journals Inc.

www.academicjournals.com

Trends in Applied Sciences Research 7 (1): 57-67, 2012
ISSN 1819-3579 / DOI: 10.3923/tasr.2012.57.67

Efficiency of Modified Adomian Decomposition for Simulating the Instability of Nano-electromechanical Switches: Comparison with the Conventional Decomposition Method

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ABSTRACT

Modeling the physical behavior of Nano-electromechanical switches (NEMS) is the center of interest for mechanical and electrical engineers. Herein, the abilities of Conventional Adomian Decomposition (CAD) and Modified Adomian Decomposition (MAD) methods in solving the governing equation of NEMS were comparatively evaluated. The pull-in instability parameters of the switch were determined and compared with those of numerical solution. It was found that using conventional decomposition method in solving NEMS problems can lead to physically incorrect results. The values of instability parameters computed by CAD series might converge to the values which differ from that obtained by numerical methods. The inaccuracy became more highlighted in the case of doubly-supported NEMS compared to cantilever one. This shortcoming was not observed for MAD and therefore, modified decomposition method could easily utilize to simulate the pull-in performance of the beam-type NEMS.

Key words: Nonlinear differential equation, conventional adomian decomposition, modified adomian decomposition, nano electromechanical switch, instability

INTRODUCTION

With increasing growth of nanotechnology, micro and nano electromechanical switches (MEMS and NEMS) have become the center of interest for researchers (Babazadeh and Keshmiri, 2009; Miskam *et al*., 2009). Many mechanical engineers have been focused on solving the nonlinear governing equation and modeling the instability of electromechanical switches. It is well-established that the governing equation of most engineering and physical systems is nonlinear in its nature. Many efforts have been conducted by scientists to solve the mathematical nonlinear equations of the systems. Recently, various mathematical methods, such as Adomian decomposition (Adomian, 1994), variational iteration (Noorzad *et al*., 2008; Shakeri *et al*., 2009; Barari *et al*., 2008), homotopy perturbation (Sharma and Methi, 2011; Fazeli *et al*., 2008) etc., have been proposed for solving nonlinear problems. Among these methods, the Adomian decomposition has been widely used to investigate engineering problems i.e. stochastic systems (Jaradat, 2008), oscillation (Momani *et al*., 2008) and heat transfer (Biazar and Amirtaimoori, 2005). After introducing the conventional Adomian method, several investigators made attempt to improve the abilities and convergence speed of the decomposition method. Rach (1984) proposed a systematic formula for computing the Adomian's polynomials. Further modification of the polynomials was also provided by Gabet (1994). Furthermore, comparison between the decomposition method and the Taylor series approximation shows that the decomposition method is much more efficient than the Taylor series method (Wazwaz, 1998). A modified Adomian decomposition method has been applied to simulate the static deflection of electrostatic micro-actuators (Kuang and Chen, 2005). Wazwaz and El-Sayed (2001) proposed a powerful modification of the Adomian decomposition method. This modification highly accelerates the convergence of the decomposition polynomials and has been applied for solving higher order boundary value problems (Wazwaz, 2000, 2001).

The aim of this paper was to evaluate the limitations/abilities of conventional and modified Adomian decomposition methods in solving constitutive equation of NEMS. In this regards, numerical solution was obtained using MAPLE commercial software and Adomian solutions were compared with the numerical results. The precision and convergence speed of both methods were compared.

GOVERNING EQUATION OF NEMS

Figure 1 shows the typical cantilever and doubly-supported beam-type NEMS constructed from a conductive electrode suspended over a conductive substrate. Applying voltage difference between the electrode and ground causes the electrode to deflect towards the ground. At a critical voltage/deflection, which is known as pull-in instability voltage/deflection, the electrode becomes unstable and pulls-in onto the substrate. The pull-in voltage and pull-in deflection of a NEMS are named as the pull-in parameters of the switches. Determining the electrode deflection and pull-in parameters of NEMS are crucial issues for engineers. Considering the van der Waals force, the governing equation of beam-type NEMS can be derived into (Ramezani *et al.*, 2008):

$$E_{eff}\, I\frac{d^4W}{dZ^4} = \frac{\varepsilon_0 dV^2}{2\,(g-W)^2}\left(1+0.65\,\frac{(g-W)}{d}\right)+\frac{Ad}{6\pi\,(g-W)^3} \tag{1a}$$

$$W\ (0) = \frac{dW\ (0)}{dZ} = 0 \text{ (B.C for cantilever and doubly-supported)} \tag{1b}$$

$$\frac{d^2W\ (0)}{dZ^2} = \frac{d^3W\ (0)}{dZ^2}\ 0 \text{ (B.C for cantilever)} \tag{1c}$$

$$W(1) = \frac{dW(1)}{dZ} = 0 \text{ (B.C for doubly-supported)} \tag{1d}$$

where, W is the deflection of the electrode, Z is the distance from the clamped end and I is the moment of inertia of the electrode cross section, E_{eff} is the effective electrode material modulus, ε_0=8.854×10^{-12} $C^2N^{-1}m^{-2}$ is the permittivity of vacuum, V is the applied voltage, g is the initial gap between the electrode and the substrate, d is the width of cross section and A is the Hamaker constant. Using the substitutions w=W/g and z=Z/L, Eq. 1 becomes:

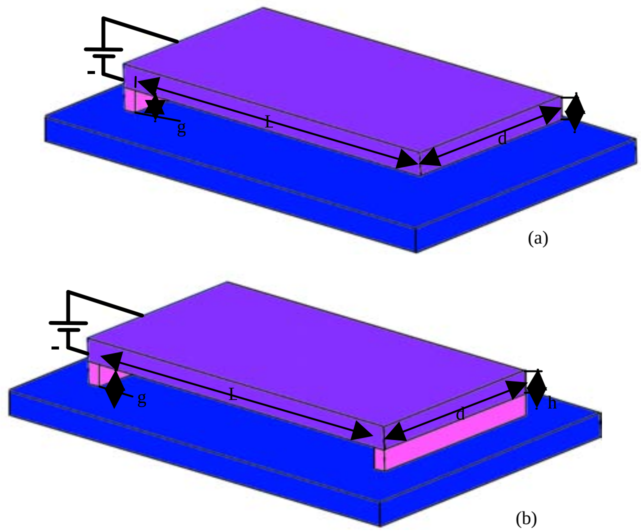

Fig. 1(a-b): (a) Schematic representation of (a) a cantilever NEMS and (b) doubly-supported NEMS

$$\frac{d^4w}{dz^4} = \frac{\alpha}{(1-w(z))^3} + \frac{\beta}{(1-w(z))^2} + \frac{\gamma\beta}{(1-w(z))} \tag{2a}$$

w (0) = 0 , w'(0) = 0 (B.C for cantilever and doubly-supported) (2b)

w''(1) = 0 , w''' (1) = 0 (B.C for cantilever) (2c)

w (1) = 0 , w'(1) = 0 (B.C for doubly-supported) (2d)

In above equations, the dimensionless parameters, α, β and γ are defined according to:

$$\alpha = \frac{AdL^4}{6\pi g^4 E_{eff} I} \tag{3a}$$

$$\beta = \frac{\varepsilon_0 dV^2 L^4}{2g^3 E_{eff} I} \tag{3b}$$

$$\gamma = 0.65\frac{g}{d} \tag{3c}$$

Using numerical computations, the variation range of above parameters which satisfies physical considerations (Ramezani *et al.*, 2008) approximately could be defined as:

- $0 \le \alpha \le 1.21$, $0 \le \beta \le 1.68$, $0 \le \gamma \le 0.65$ For cantilever NEMS
- $0 \le \alpha \le 50.09$, $0 \le \beta \le 70.06$, $0 \le \gamma \le 0.65$ For doubly-supported NEMS

Note that at the onset of the instability, the maximum deflection of the electrode increases without requiring any further increase in voltage. In mathematical view, the slope of w-β curve reaches infinity when instability occurs, i.e., $dw/d\beta(z = 1) \to \infty$ and $dw/d\beta(z = 0.5) \to \infty$ for cantilever and doubly-supported NEMS, respectively. As a convenient approach, the pull-in instability voltage, β_{PI} and pull-in deflection, u_{PI}, of NEMS can be determined via plotting w(z = 1) vs. β for cantilever and w (z = 0.5) vs. β for doubly-supported NEMS.

FUNDAMENTALS OF DECOMPOSITION METHODS

Consider a differential equation of a fourth-order boundary-value problem (Wazwaz, 2001):

$$y^{(4)}\ (x) = f\ (x, y),\ \ 0 \le x \le L_b \tag{4}$$

With boundary conditions:

$$y(0) = \alpha_0,\ \ y'(0) = \alpha_1 \tag{5}$$

Equation 4 can be represented as:

$$L^{(4)}\left[y(x)\right] = f(x, y) \tag{6}$$

Where, $L^{(4)}$ is a differential operator which is defined as:

$$L^{(4)} = \frac{d^{(4)}}{dx^{(4)}} \tag{7}$$

The corresponding inverse operator $L^{-(4)}$ is defined as a 4-fold integral operator, that is:

$$L^{-(4)} = \int_0^x \int_0^x \int_0^x \int_0^x (\ \ )\, dx\, dx\, dx\, dx \tag{8}$$

Employing the decomposition method (Wazwaz, 2001), the dependent variable in Eq. 4 can be written as:

$$y(x) = \sum_{n=0}^{\infty} y_n(x) = \alpha_0 + \alpha_1 x + \frac{1}{2} C_1 x^2 + \frac{1}{3!} C_2 x^3 + L^{-(4)}\left[\sum_{n=0}^{\infty} A_n\right] \tag{9}$$

where, constants C_1 and C_2 can be determined from the boundary condition at another boundary point. In above relations, function A_n approximates nonlinear function f (x,y) and is determined as a polynomial series:

$$f(x,y)=\sum_{n=0}^{\infty}A_n \tag{10}$$

According to Conventional Adomian Decomposition (CAD), series A_n is obtained using the following formula:

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f(\lambda)\right]_{\lambda=0} \tag{11}$$

On the other hands, according to Modified Adomian Decomposition (MAD), the following convenient equations can be utilized to obtain an appropriate solution for A_n (Adomian, 1986; Rach, 1984):

$$A_n=\sum_{v=1}^{n}C(v,n)\frac{d^v}{d\lambda^v}\left[f(\lambda)\right]_{\lambda=0} \tag{12}$$

where,

$$C(n,v)=\Sigma_{p_i}\Pi_{i=1}^{v}\left(1/k_i!\right)f_{p_i}^{k_i}\ ,\ \sum_{i=1}^{v}k_ip_i=n\ \ n>0,$$

$0\le i\le n$, $1\le p_i\le n$,-v+1 and k_i is the number of repetition of the fp_i, the values of p_i are selected from the above range by combination without repetition.

Now, according to decomposition methods, the recursive relations of Eq. 9 can be provided as follows:

$$\begin{aligned}&y_0(x)=\alpha_0,\ y_1(x)=\alpha_1x+\frac{1}{2}C_1x^2+\frac{1}{3!}C_2x^3+L^{-(4)}\left[A_0\right],\\&y_{k+1}(x)\ L^{-(4)}\left[A_k\right],\ k\ge 1\end{aligned} \tag{13}$$

In order to apply decomposition methods for simulating deflection and pull-in behavior of NEMS, the substitution y=1-w is used to rewrite Eq. 2 into the following form:

$$\frac{d^4y}{dz}=-\frac{\alpha}{y(z)^3}-\frac{\beta}{y(z)^2}-\frac{\gamma\beta}{y(z)} \tag{14a}$$

$$y(0)=1\ ,\ y'(0)=0\ \text{(B.C for cantilever and doubly-supported)} \tag{14b}$$

$$y''(1)=1\ ,\ y'''(1)=0\ \text{(B.C for cantilever NEMS)} \tag{14c}$$

$$y(1)=0\ ,\ y'(1)=0\ \text{(B.C for doubly-supported NEMS)} \tag{14d}$$

The solution of Eq. 2 can be represented as:

$$y(z)=\sum_{n=0}^{\infty}y_n=1+\frac{C_1z^2}{2!}+\frac{C_2z^3}{3!}-L^{-(4)}\left[\alpha\sum_{n=0}^{\infty}A_{n,3}+\beta\sum_{n=0}^{\infty}A_{n,2}+\beta\gamma\sum_{n=0}^{\infty}A_{n,1}\right] \tag{15}$$

where, the constants C_1 and C_2 can be determined by solving the resulted algebraic equations from B.C. at z = 1, i.e., using Eq. 14c and 14d for cantilever and doubly-supported NEMS, respectively.

Conventional Adomian method (CAD): In order to solve Eq. 15 using CAD, formula (11) is expanded to obtain:

$$\begin{aligned}&A_{0,m}=y_0^{-m},\ A_{1,m}=-my_0^{-m-1}y_1,\ A_{2,m}=\frac{1}{2}m(m+1)y_0^{-m-2}y_1^2-my_0^{-m-1}y_2,\\&A_{3,m}=-\frac{1}{6}m(m+1)(m+2)y_0^{-m-3}y_1^3+m(m+1)y_0^{-m-2}y_1y_2-my_0^{-m-1}y_3,\\&\vdots\ =\ \vdots\end{aligned} \tag{16}$$

Substituting Eq. 16 in recursive Eq. 13, we obtain:

$$\begin{aligned}&y_0=1,\ y_1=0,\ y_2=\frac{C_1z^2}{2},\ y_3=\frac{C_2z^3}{6},\ y_4=-(\alpha+\beta+\gamma\beta)\frac{z^4}{24},\ y_5=0,\\&y_6=C_1(3\alpha+2\beta+\gamma\beta)\frac{z^6}{720},\ y_7=C_2(3\alpha+2\beta+\gamma\beta)\frac{z^7}{5040},\\&y_8=-\left[C_1^2(6\alpha+3\beta+\gamma\beta)+\frac{(3\alpha+2\beta+\gamma\beta)(\alpha+\beta+\gamma\beta)}{6}\right]\frac{z^8}{6720},\\&\ldots\end{aligned} \tag{17}$$

Therefore the solution of Eq. 2 is obtained as:

$$\begin{aligned}\frac{W}{g}(z)=&-\frac{C_1z^2}{2!}-\frac{C_1z^3}{3!}+(\alpha+\beta+\lambda\beta)\frac{z^4}{4!}-(3\alpha+2\beta+\gamma\beta)\frac{Az^6}{6!}\\&-(3\alpha+2\beta+\lambda\beta)\frac{Bz^7}{7!}+\left[A^2(6\alpha+3\beta+\gamma\beta)+\frac{(3\alpha+2\beta+\gamma\beta)(\alpha+\beta+\gamma\beta)}{6}\right]\frac{z^8}{8!}+\cdots\end{aligned} \tag{18}$$

Modified Adomian method (MAD): In the case of modified domain methods (Eq. 12), it is obtained:

$$\begin{aligned}&A_{0,m}=\frac{1}{y_0^{m}},\ A_{1,m}=y_1\left(\frac{1}{y_0^{m}}\right),\ A_{2,m)}=y_2\left(\frac{1}{y_0^{m}}\right)'+\frac{1}{2!}y_1^2\left(\frac{1}{y_0^{m}}\right)'',\\&A_{3,m}=y_3\left(\frac{1}{y_0^{m}}\right)'+y_1y_2\left(\frac{1}{y_0^{m}}\right)''+\frac{1}{3!}y_1^3\left(\frac{1}{y_0^{m}}\right)''',\ldots\ldots\end{aligned} \tag{19}$$

Substituting Eq. 19 in Eq. 13, we obtain:

$$
\begin{aligned}
&y_0 = 1,\ y_1 = \frac{1}{2!} C_1 z^2 + \frac{1}{3!} C_2 z^3 - \frac{1}{4!} (\alpha + \beta + \beta\gamma)\ z^4, \\
&y_2 = (3\alpha + 2\beta + \beta\gamma)\ (\frac{1}{6!} C_1 z^6 + \frac{1}{7!} C_2 z^7 - \frac{1}{8!} (\alpha + \beta + \beta\gamma)\ z^8) \\
&y_3 = -\frac{C_1^2}{8!} (36\alpha + 18\beta + 6\beta\gamma)\ z^8 - \frac{C_1 C_2}{9!} (120\alpha + 60\beta + 20\beta\gamma)\ z^9 \\
&+ \frac{1}{10!} \left[(3\alpha + 2\beta + \beta\gamma)^2\ C_1 + (\alpha + \beta + \beta\gamma)\ (6\alpha + 3\beta + \beta\gamma)\ (30C_1 - 20C_2^2)\right] z^{10} \\
&+ \frac{1}{11!} \left[(3\alpha + 2\beta + \beta\gamma)^2\ C_2 + 70C_2\ (\alpha + \beta + \beta\gamma)\ (6\alpha + 3\beta + \beta\gamma)\right] z^{11}
\end{aligned} \tag{20}
$$

Therefore, the solution of Eq. 2 can be summarized to:

$$
\begin{aligned}
&\frac{W}{g}(z) = -\frac{1}{2!} C_1 z^2 - \frac{1}{3!} C_2 z^3 + \frac{1}{4!} (\alpha + \beta + \beta\gamma) z^4 - \frac{C_1}{6!} (3\alpha + 2\beta + \beta\gamma) z^6 - \frac{C_2}{7!} (3\alpha + 2\beta + \beta\gamma) z^7 \\
&+ \frac{1}{8!}\left[6C_1^2(6\alpha + 3\beta + \beta\gamma) + (\alpha + \beta + \beta\gamma)(3\alpha + 2\beta + \beta\gamma)\right] z^8 + \frac{20C_1C_2}{9!}(6\alpha + 3\beta + \beta\gamma) z^9 \\
&- \frac{1}{10!}\left[(3\alpha + 2\beta + \beta\gamma)^2 C_1 + (\alpha + \beta + \beta\gamma)(6\alpha + 3\beta + \beta\gamma)(30C_1 - 20C_2^2)\right] z^{10} \\
&- \frac{1}{11!}\left[(3\alpha + 2\beta + \beta\gamma)^2 C_2 + 70C_2(\alpha + \beta + \beta\gamma)(6\alpha + 3\beta + \beta\lambda)\right] z^{11} + \ldots
\end{aligned} \tag{21}
$$

Case studies and comparing of the methods: In order to compare decomposition methods, typical cantilever and a doubly-supported NEMS are simulated and the results are compared with numerical data.

Figure 2a-c shows the variation of tip deflection as a function of series terms for three typical cantilever NEMS ($\beta = \gamma = 0.5$) with different van der Waals coefficients (Fig. 2a-c) correspond to a = 0, 0.25 and 0.5, respectively. This figure reveals that the value of α coefficient has a great influence on the convergence of the conventional series. As seen, the CAD series might not converge for large α values (Fig. 2a). However, this shortcoming is not observed in the case of the MAD series (Fig. 2d-f) where the series solution rapidly converges to the numerical solution. Figure 3 shows the convergence of pull-in value for typical cantilever NEMS obtained by various series terms. This figure reveals that CAD converges to a pull-in value which is different from numerical values. However, the pull-in value obtained by modified method converges to that of the numerical value. Figure 4 shows the variation of pull-in voltage for cantilever NEMS as a function of van der Waals force parameter (α). This figure shows that the difference between Adomian and numerical solutions increases by increasing the a value. As seen, no solution exist, when α exceeds its critical value.

Figure 5 shows the variation of tip deflection for typical doubly-supported NEMS (a =β = 5, γ = 0.5) as a function of series terms. This figure reveals that conventional decomposition can not be applied for modeling pull-in performance of doubly-supported NEMS. As seen, while

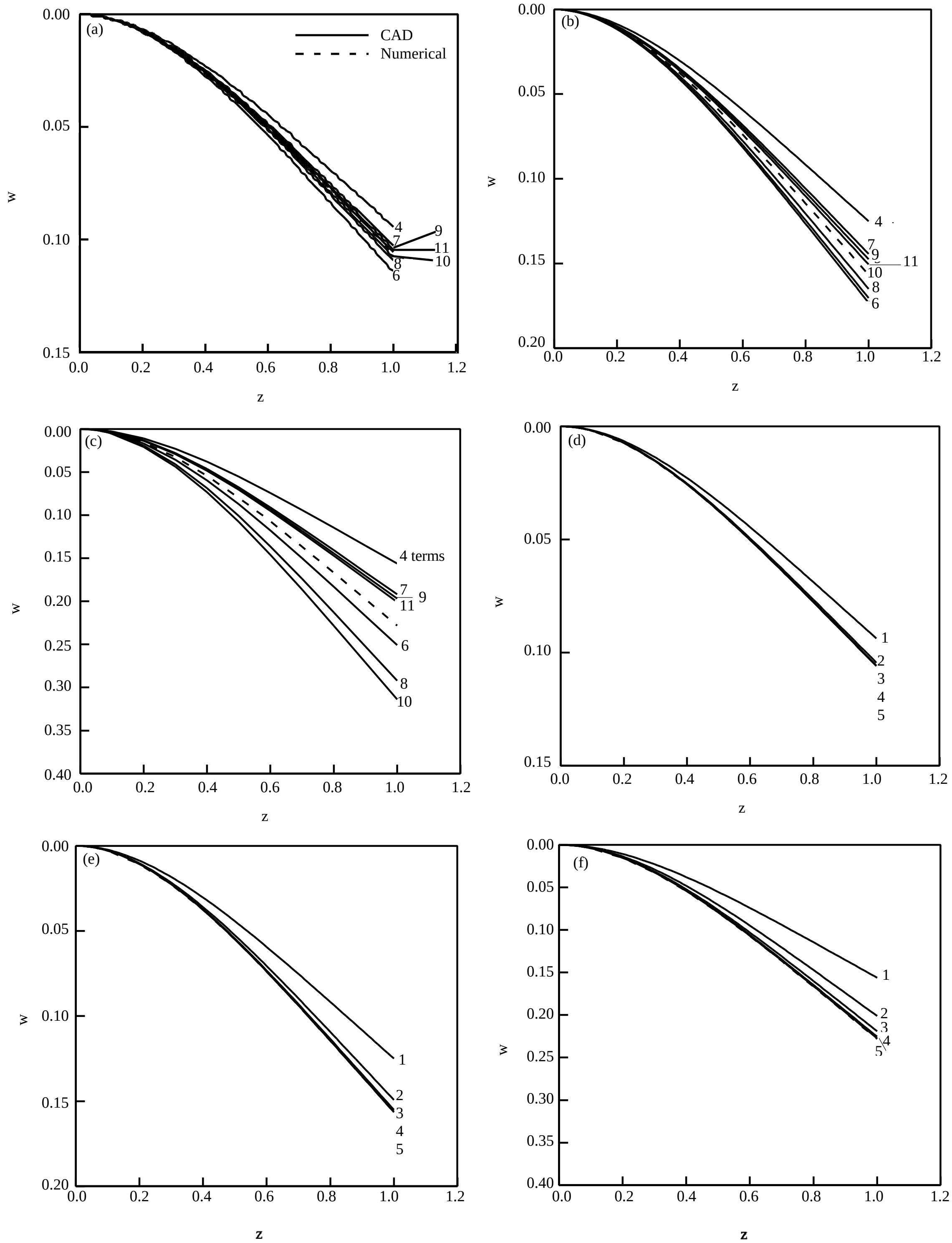

Fig. 2 (a-f): Convergence check for the NEMS tip deflection ($\beta = \gamma = 0.5$) vs, number of seies terms for three typical cantilever cases: (a) and (d) $\alpha = 0$, (b) and (e) $\alpha = 0.25$ (c) and (f) $\alpha = 0.5$

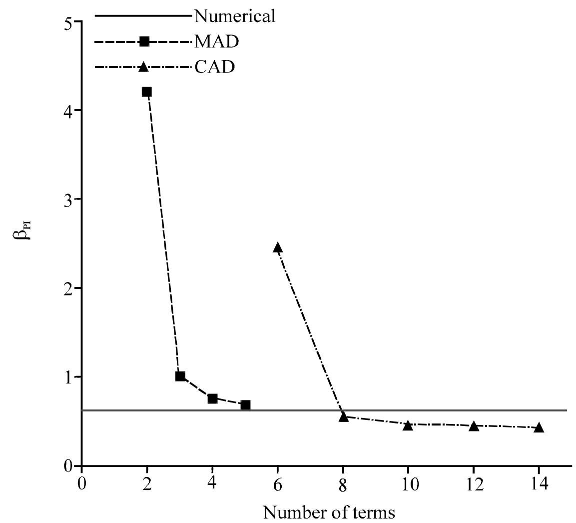

Fig. 3: Variation of β_{PI} for typical cantilever NEMS (α = 0.5 and γ = 0.65)

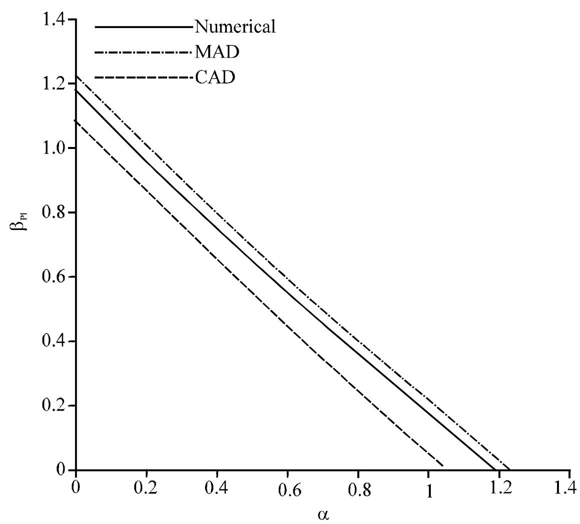

Fig. 4: Variation of pull-in voltage (β_{PI}) of cantilever NEMS as a function of van der Waals force (a) (γ = 0.65)

the MAD method rapidly converges to the numerical solution, CAD series converges to an unacceptable value. Furthermore, Table 1 shows the convergence of pull-in voltage of typical doubly-supported NEMS obtained by Adomian method using various series terms. As seen β_{PI} values obtained by MAD series converge to that of numerical value, i.e., β_{PI} = 43.575. In Table 1, only the β_{PI} values obtained by MAD have been presented since the CAD method is not reliable for simulating double-supported NEMS. Note that the MAD series which are not able to capture the instability of the switch are physically meaningless and cannot be used for investigating the pull-in performance of the NEMS.

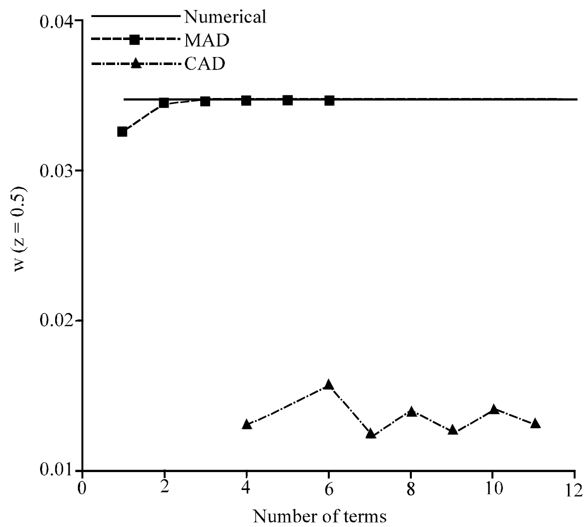

Fig. 5: Convergence check for the tip deflection of a typical doubly-supported NEMS ($\alpha = \beta = 5$, $\gamma = 0.5$) vs. number of series terms

Table 1: Convergence check of pull-in voltage for typical NEMS (α = 5 and γ = 0.65). As seen, β_{PI} values obtained by Adomian series converge to that of numerical value (i.e., β_{PI} = 43.575)

	2 Terms	3 Terms	4 Terms	5 Terms	6 Terms
Value of β_{PI} obtained by modified adomian	Can't determine pull-in	61.701	Can't determine pull-in	45.298	Can't determine pull-in
Difference with Numerical (%)	Can't determine pull-in	41.60	Can't determine pull-in	3.95	Can't determine pull-in

CONCLUSION

Modified and conventional Adomian decomposition methods were applied to solve nonlinear governing equation of beam-type NEMS. The deflection and pull-in parameters of cantilever and doubly-supported NEMS were computed and the result was compared with the numerical solution. It was observed that the modified Adomian method provides accurate results and converges rapidly to numerical solution. However, the convergence of the conventional series highly depends on the values of constant coefficients in the NEMS governing equation. Interestingly, we showed that there are some cases that using CAD method could lead to physically incorrect results. Specially, for doubly-supported NEMS, the deflection value computed by conventional decomposition series might be very different from that of numerical method. Interestingly, none of these shortcomings was observed for modified Adomian decomposition series.

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