



Trends in
**Applied Sciences
Research**

ISSN 1819-3579



Academic
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Adaptive Backstepping Control of a New Chaotic System in the Presence of Disturbance

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ABSTRACT

In this study, an adaptive backstepping design is proposed to control a new chaotic system in the presence of disturbance. The control input stabilizes the origin that is unstable. Adaptation laws for the parameters are derived and implemented. Numerical simulations show the effectiveness and feasibility of the method.

Key words: Adaptive backstepping, chaos, chaotic system

INTRODUCTION

The chaos control has been an attractive field of research in the last two decades. Today, chaos is seen to have multiple useful applications in physics, chemistry, biology, ecology and many engineering systems such as in chemical reactors, power converters, information processing and secure communications (Chen, 1999; Chen and Dong, 1993, 1998; Kapitaniak, 1996; Lakshmanan and Murali, 1996; Boccaletti *et al.*, 2000). The control of chaotic systems can be divided into two tasks; first we should get rid of the chaotic behavior and at the second state the system should be stabilized at one of its equilibrium points (Liao and Yu, 2006). OGY was the first proposed chaos control method, composed of the first characters of its originators (Ott *et al.*, 1990).

Until now, many advanced theories and methods have been proposed for controlling chaos, such as state feedback control (Chen and Lu, 2003), robust control (Kameda *et al.*, 1991; Lu *et al.*, 2003), adaptive control (Zeng and Singh, 1997; Bernardo, 1996; Feki, 2003; Wu *et al.*, 1996; Changchun and Xinping, 2004), optimal control (Piccardi and Ghezzi, 1997), structure variation control (Yu, 1996), fuzzy control (Jiang *et al.*, 2005) etc.

The backstepping approach is one of the most popular nonlinear techniques for nonlinear control design. The technique is a systematic design approach and consists in a recursive procedure that skillfully constructs a Lyapunov function and design the control input (Lu and Zhang, 2001; Park, 2006; Bowong and Moukam Kakmeni, 2006; Yassen, 2006). The adaptive backstepping is a novel approach for controlling the uncertain nonlinear systems for most of the strict feedback systems (Wang and Ge, 2001; Yu and Zhang, 2004). Guo-Liang *et al.* (2007) presented a new chaotic dynamical system with three state variables as follows:

$$\begin{aligned}\dot{x} &= a(y-x) \\ \dot{y} &= bx + cy - xz \\ \dot{z} &= x^2 - hz\end{aligned}\tag{1}$$

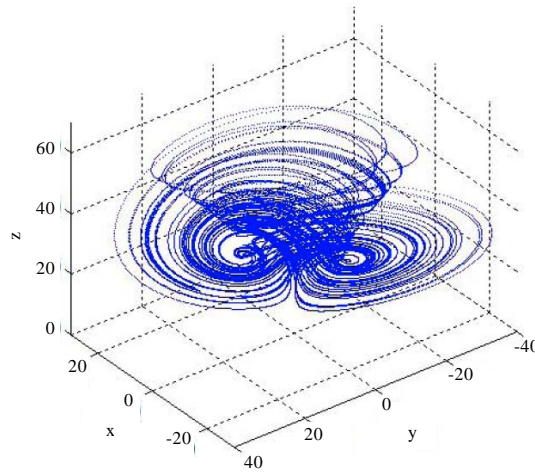


Fig. 1: Chaotic attractor of system (1)

where, a , b , c and h are system parameters. It is clear that system (1) has three unstable equilibrium points $0_1(0,0,0)$:

$$0_2(\sqrt{h(b+c)}, \sqrt{h(b+c)}, b+c)$$

and:

$$0_2(-\sqrt{h(b+c)}, -\sqrt{h(b+c)}, b+c)$$

System (1) has a chaotic attractor as shown in Fig. 1 when $q = 20$, $b = 14$, $c = 10.6$ and $h = 2.8$.

Zhu (2009) used three feedback control methods on the basis of the linearization to stabilize the unstable equilibrium points of this system. In the present study, a parametric model of this system is considered and by using adaptive backstepping control algorithm the origin that is an equilibrium point of this system is stabilized. The controller is designed in such a way that the system has internal dynamics and it is shown that it is stable. The simulation results show that the system is stable and the parameters of the system converge to fixed values, from the two sets of initial conditions. But they don't converge to the same values in two simulations.

Adaptive backstepping control: Consider the controlled new chaotic system as follows:

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= bx + cy - xz + d + u \\ \dot{z} &= x^2 - hz \end{aligned} \tag{2}$$

where, a, b, c and h are four unknown constant parameters, d is the bounded disturbance and u is a controller. In the following we will find a controller u and parameter adaptation laws for the uncertain parameters of the system (2) in order to stabilize the origin. The adaptive backstepping procedure includes three steps.

Step 1: Let $z_1 = x$ and consider the following Lyapunov function:

$$V_1(t) = \frac{1}{2} z_1^2 \tag{3}$$

The time derivative of $V_1(t)$ is:

$$\dot{V}_1(t) = z_1 (a(z_1 - y)) \tag{4}$$

Now let y be a virtual control input, it should be defined such that. Assume that $u = a_1 = px$ where $p < 1$ then by assuming $a > 0$ it is obvious that:

Step 2: Define the following variables:

$$\begin{aligned} z_2 &= y - a_1 \\ \tilde{a} &= a - a_1 \\ \tilde{b} &= b - b_1 \\ \tilde{c} &= c - c_1 \\ \tilde{d} &= d - d_1 \end{aligned} \tag{5}$$

where a_1, b_1, c_1, d_1 are the estimations of a, b, c and d , respectively.

Choose Lyapunov function as:

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2} \tilde{a}^2 + \frac{1}{2} \tilde{b}^2 + \frac{1}{2} \tilde{c}^2 + \frac{1}{2} \tilde{d}^2 \tag{6}$$

Its time derivative is:

$$\begin{aligned} \dot{V}_2 &= z_2^1 a(p-1) - \tilde{a}\dot{a}_1 - \tilde{b}\dot{b}_1 - \tilde{c}\dot{c}_1 - \tilde{d}\dot{d}_1 + \\ & z_2 ((a+b)x + cy - xz = pa(y-x+d+u)) \end{aligned} \tag{7}$$

Now the control input u and the adaptive update laws for a_1, b_1, c_1, d_1 should be designed such that. One can choose them as:

$$u = xz - (a_1 + b_1 - p)x - (c_1 + 1)y + pa_1(y - x) - d_1 \tag{8}$$

$$\dot{a}_1 = [(p+1)x - py]z_2 \tag{9}$$

$$\dot{b}_1 = xz_2 \tag{10}$$

$$\dot{c}_1 = yz_2 \tag{11}$$

$$\dot{d}_1 = z_2 \tag{12}$$

As a result, Eq. 2 would become:

$$\dot{V}_2 = z_1^2 a(p-1) - z_2^2 \tag{13}$$

It is clear that $\dot{V}_2 \leq 0$. Therefore, it is concluded that the state variables z_1 and z_2 converge to zero. Hence, using Lasalle-Yoshizawa theorem it can be shown that z_1 and z_2 converges asymptotically to zero. By the definition of these variables it is understood that x and y tend to zero asymptotically. Considering the third equation of the system (2) reveals that as x and y tend to zero, z is also converges to zero and the zero dynamics of the system is stable. As a result the equilibrium point $0_1(0,0,0)$ of the system is globally asymptotically stable. Therefore, the following theorem is proved:

Theorem: Consider the system (2), using the update laws (9-12) for the parameters and the disturbance and the controller (8); the system (2) is globally asymptotically stable.

Proof: Considering the radially unbounded Lyapunov function (6), it was shown that x and y tend to zero, also the internal dynamics of the system that is the third equation of (2) is stable by the above description, so the system (2) is globally asymptotically stable.

Numerical simulations: Here, the effectiveness of the proposed controller and adaptation laws is verified via numerical simulations. Let $a = 20$, $b = 14$, $c = 10.6$ and $h = 2.8$ be the nominal values of the parameters and assume the disturbance is $d = 2\sin(t)$ for the first simulation and for the second one $d = 4\sin(2t)$. Two sets of initial conditions have been considered and the simulation results illustrate the performance of the designed controller. The initial conditions are shown in Table 1. The simulation results are illustrated in Fig. 2-5. Figure 2 and 4 show that the

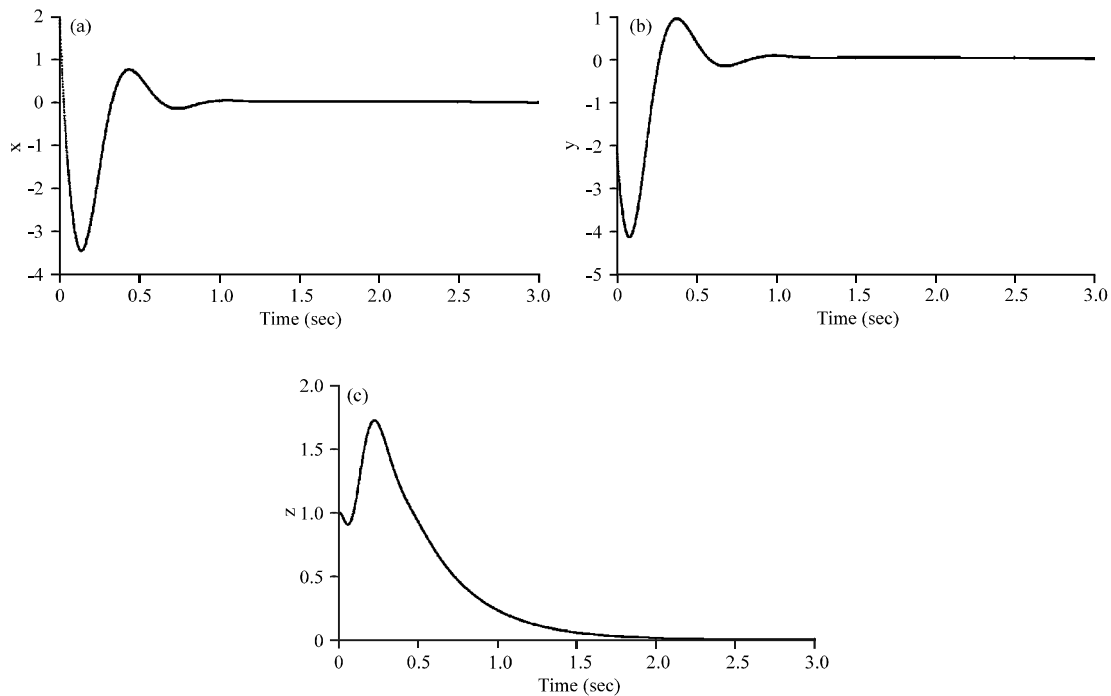


Fig. 2(a-c): State trajectories of closed loop system given in Eq. 2 with the first set of initial conditions, (a) State x, (b) State y and (c) State z

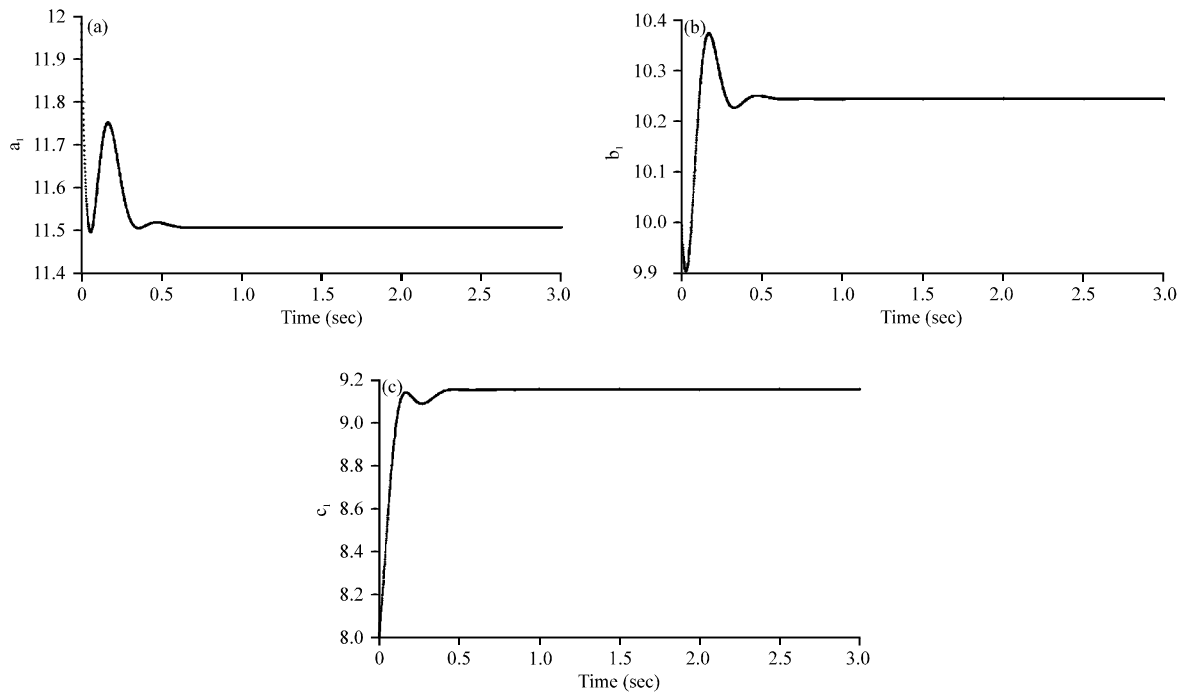


Fig. 3(a-c): Parameters of closed loop system given in Eq. 2 with the first set of initial conditions, (a) Parameter a_1 , (b) Parameter b_1 and (c) Parameter c_1

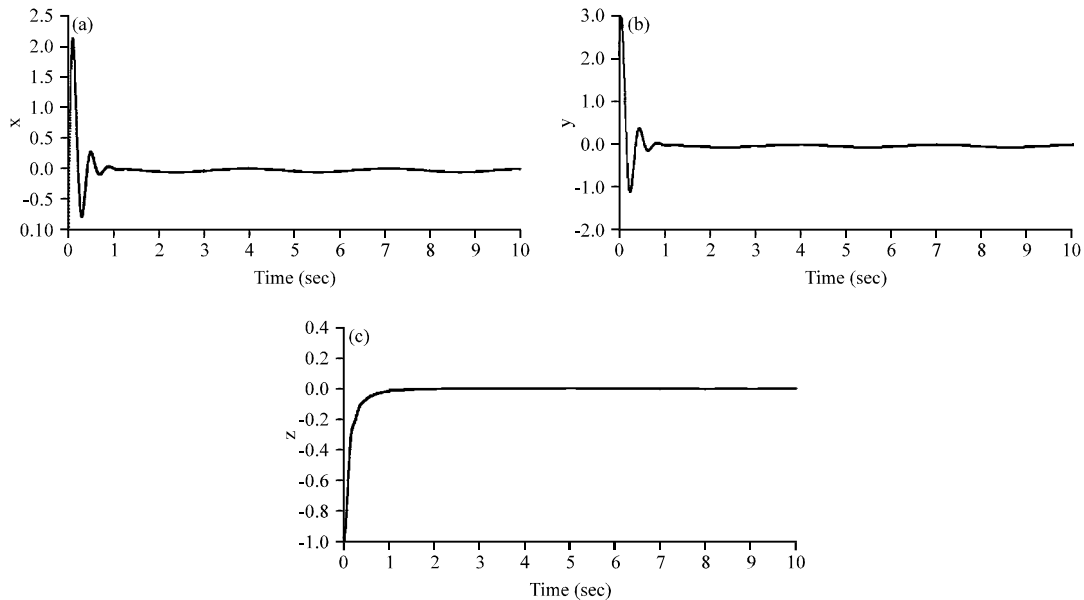


Fig. 4(a-c): State trajectories of closed loop system given in Eq. 2 with the second set of initial conditions, (a) State x , (b) State y and (c) State z

states of the system tend to zero and Fig. 3 and 5 demonstrate that the parameters of the system converge to some constant values.

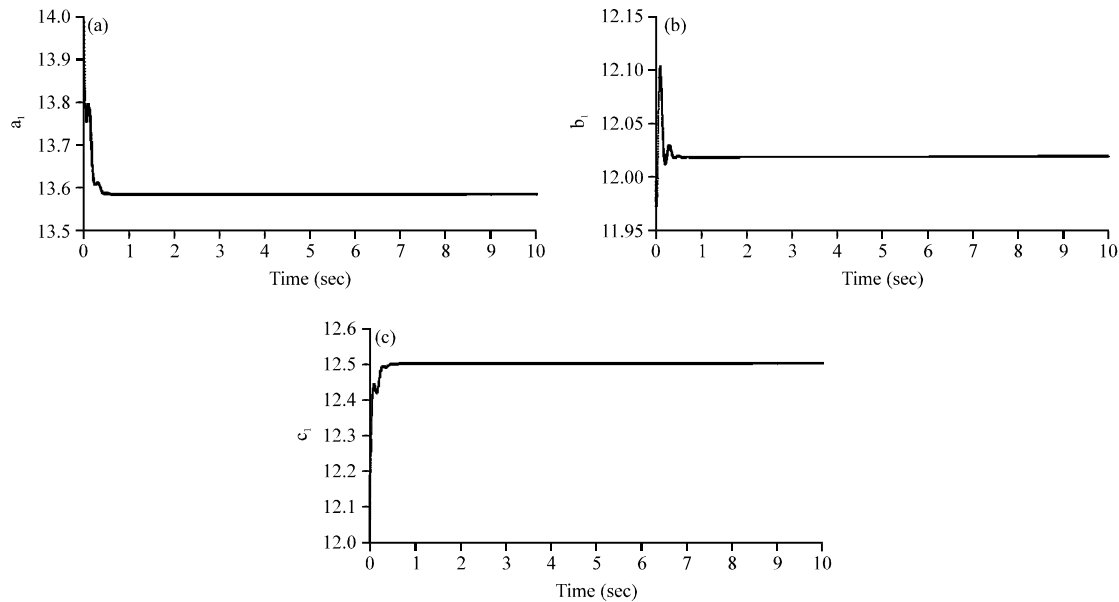


Fig. 5(a-c): State trajectories of closed loop system given in Eq. 2 with the second set of initial conditions, (a) State x, (b) State y and (c) State z

Table 1: Sets of initial conditions

Set 1	Set 2
The state variables	The state variables
$[2-21]^T$	$[-12-1]^T$
$a_1(0) = 12$	$a_1(0) = 14$
$b_1(0) = 10$	$b_1(0) = 12$
$c_1(0) = 8$	$c_1(0) = 12$
$d_1(0) = 0.15$	$d_1(0) = 0.3$

CONCLUSIONS

In this study, adaptive backstepping design has been proposed to control a new chaotic system. There exists parameter uncertainty and the disturbance. All the results were obtained based on Lyapunov stability theorem and Lasalle-Yoshizawa theorem. Numerical simulations were given to validate the proposed approach.

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