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Mathematical Analysis on Pulse Width Modulated Switching Functions of Matrix Converter

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ABSTRACT

The objective of this study was to present a set of analytical equations for the purpose of computing harmonics of pulse width modulated switching functions, for a three phase matrix converter. The analytical technique for derivation of mathematical equations is based on the two dimensional or Double Fourier Series (DFS) concept. For the sake of simplicity, another method is proposed for analyzing Pulse Width Modulation (PWM) harmonic spectrum. It is shown that the whole process will be greatly simplified by employing Fourier series of a pulse train. Computations of harmonic components of switching functions are carried out on the basis of the proposed analytical technique and compared with the former technique based on DFS. The different ways of creating switching functions are also studied.

Key words: Double Fourier series, Fourier analysis, harmonic analysis, matrix converter, pulse width modulation, switching function

INTRODUCTION

Three-phase matrix converter is a direct Alternating Current (AC) to AC power converter with no Direct Current (DC) link (Wheeler *et al.*, 2002). The schematic circuit is shown in Fig. 1. Recent growing interest in modeling and analysis of power electronic circuits include using switching functions (Marouchos, 2006; Marouchos *et al.*, 2010). In this regard it is causally necessary to find expressions of switching functions in the analysis of switching power converters. The relation of the input voltages (currents) to the output voltages (currents) provides the basis for finding the solution to the switching functions. Alesina and Venturini (1981) came up with the complete solution to the switching functions of three-phase matrix converter. These solutions are expressed in the sinusoidal forms. However, in practical implementations, power switches only get the values of ones (the switch is turned on) and zeros (the switch is turned off). The established techniques which are usually employed to enable to generate the switching functions for power converters are PWM based. One of our interests in this study is in determining how the sinusoidal forms of switching functions can be generated with a view to using PWM technique. Although this method is well-known in control of AC to DC power converters, very few articles have described the detailed implementation of this, for matrix converter. In this regard three different ways of producing pulses for switching functions will be explored. In the first method equal pulses are used in each switching period. The second and the third methods are based on the PWM technique. Implementation of the

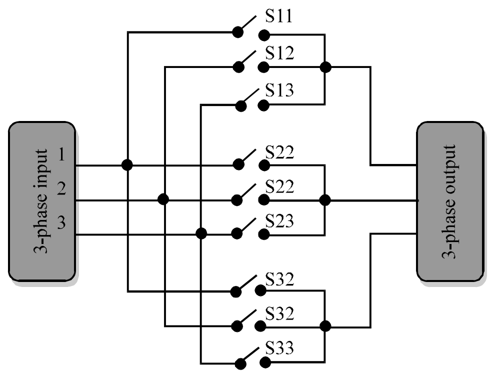

Fig. 1: Three-phase matrix converter with nine (s_{ij}) switches matrix converter

second method is based on the triangular carrier comparison process. In the third method a digital PWM process will be studied. The procedure of creating pulses is fully described. PWM is a well-established technique with non-linear characteristics. There is a demanding requirement for spectral analysis where PWM process is put to use. In previous studies (Mirkazemi-Moud *et al*., 1994; Shen *et al*., 1997; Deslauriers *et al*., 2005), a double Fourier series expansion has been used to obtain a theoretical solution to the harmonic spectrum of the PWM signals for AC to DC converters. The chief aim of this study is to apply DFS to the PWM switching functions of a matrix converter and give the exact analytical expressions for them. Alesina and Venturini (1981) applied a general Fourier transform to model the switches of matrix converter. However, the statements in the study are rather complex and there is not a closed form solution to analyze harmonic components. A linearized analysis is employed (Casadei *et al*., 1998) to determine the matrix converter performance neglecting the effects of the switching harmonics. In some papers, matrix converter operation has been simulated on digital computers (using FFT toolboxes) in order to obtain the harmonic spectrum (Kim *et al*., 2010; Lou and Pan, 2006). As far as we know there is no report on evaluation of harmonic components based on DFS for modulated switching functions of a matrix converter. DFS is the basic and established technique to obtain the spectrum of PWM switching functions but not a simple one. Undergraduate students may not be familiar with DFS concept and find themselves confronted by complexity of computing two dimensional Fourier series coefficients and Bessel functions which appear in double Fourier series coefficients. Extension of the double Fourier series is messy and finding the spectrum of the signals analytically, is somewhat complicated. In this regard, there is a need for a simple and accurate mathematical method that effectively eliminates complexity of the problem. The secondary goal of this study was to give analytical expressions for PWM spectrum of switching functions using the familiar one-dimensional Fourier series of a pulse train. This analytical approach eliminates the need for computation of DFS coefficients and Bessel functions. The assumption of $f_m \ll f_{switching}$ (f_m: modulation frequency; and carrier or switching frequency), which is acceptable in PWM process, will be used in this method. This results in a considerable simplification of the PWM spectrum analysis.

MATRIX CONVERTER FUNDAMENTALS AND CONSTRAINTS

For a three-phase matrix converter let us assume an input voltage set like (Eq. 1):

$$\left[V_i(t)\right] = \begin{bmatrix} V_{im}\cos(\omega_i t) \\ V_{im}\cos(\omega_i t + \frac{2\pi}{3}) \\ V_{im}\cos(\omega_i t + \frac{4\pi}{3}) \end{bmatrix} \tag{1}$$

It is generally assumed that the desired three-phase output voltage is like (Eq. 2). The meaning behind these equations is that the input and output voltages may have different frequencies, voltage amplitudes and phase angles:

$$\left[V_o(t)\right] = \begin{bmatrix} qV_{im}\cos(\omega_o t + \psi) \\ qV_{im}\cos(\omega_o t + \frac{2\pi}{3} + \psi) \\ qV_{im}\cos(\omega_o t + \frac{4\pi}{3} + \psi) \end{bmatrix} \tag{2}$$

Hence, the relationship between the input voltages and the output voltages can be written as (Eq. 3):

$$\begin{gathered} \left[V_o(t)\right] = \left[S(t)\right] \times \left[V_i(t)\right] \\ \left[V_o(t)\right] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} . \left[V_i(t)\right] \end{gathered} \tag{3}$$

Similarly the relationship between the input and output currents is like (Eq. 4):

$$[I(t)_{in}] = [S(t)]^T \times [V_i(t)] \tag{4}$$

The 3×3 matrix [S], determines the relation of output and input quantities and is the basis of finding the solution for modulation problem. In matrix [S], s_{ij} represents the switch state at the specific time.

Each s_{ij} is zero when the switch is off and is 1 when the corresponding switch is conducting. Equation 5 demonstrates a hypothetical example of matrix [S(t)] at specific time t:

$$S(t_\gamma) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{5}$$

Considering the matrix converter shown in Fig. 1, the constraint equations for switching matrix, [S(t)], can be determined by using the circuit theory. The restriction imposed by Kirchhoff's Voltage Law (KVL) can be written in mathematical expressions. For [S(t)] KVL means that the column sum is not allowed to go beyond one:

$$S_{11}(t)+S_{21}(t)+S_{31}(t)\le 1 \tag{6}$$

In other words, the input terminals should not be short circuited. On the other hand the restriction imposed by Kirchhoff's Current Law (KCL) should be taken into consideration. For [S(t)] it means that at least one switch in each column should be ON at any time, i.e. the sum of any column should be equal or greater than 1:

$$S_{11}(t)+S_{21}(t)+S_{31}(t)\ge 1 \tag{7}$$

This is required for the operation of matrix converter since the output phases are not allowed to be opened. When the KVL and KCL restrictions are combined, the mathematical expression for matrix converter constraint can be written as:

$$\begin{aligned} &S_{1j}(t)+S_{2j}(t)+S_{3j}(t)=1 \\ &j=1,\ 2,\ 3 \end{aligned} \tag{8}$$

As already mentioned, the modulation problem is to find the proper solutions for [S(t)], which satisfy the constraint equations. Two solutions were found by Alesina and Venturini (1981) for [S(t)]. The solutions are expressed in Eq. 9 and 10. A linear combination of both solutions may also be used. In this study an input voltage with frequency of 60 Hz and an output voltage with the same frequency are assumed. The procedure of creating pulses is discussed for the second solution, i.e., Eq. 10. Practically the same procedure can be applied for implementation of the first solution or any linear combination of the two solutions. Note that in Eq. 9 and 10:

$$f_m=\frac{\omega_m}{2\pi}$$

is called the modulation frequency and q is the ratio of the output voltage amplitude to the input voltage amplitude:

$$[S_i(t)]=\frac{1}{3}\begin{bmatrix} 1+2q\cos(\omega_m t) & 1+2q\cos(\omega_m t-\frac{2\pi}{3}) & 1+2q\cos(\omega_m t-\frac{4\pi}{3}) \\ 1+2q\cos(\omega_m t-\frac{4\pi}{3}) & 1+2q\cos(\omega_m t) & 1+2q\cos(\omega_m t-\frac{2\pi}{3}) \\ 1+2q\cos(\omega_m t-\frac{2\pi}{3}) & 1+2q\cos(\omega_m t-\frac{4\pi}{3}) & 1+2q\cos(\omega_m t) \end{bmatrix} \tag{9}$$

$$\begin{aligned} &\text{for } \omega_m=\omega_o-\omega_i \\ &\text{and } 0\le q\le 0.5 \end{aligned}$$

$$[S_i(t)]=\frac{1}{3}\begin{bmatrix} 1+2q\cos(\omega_m t) & 1+2q\cos(\omega_m t-\frac{2\pi}{3}) & 1+2q\cos(\omega_m t-\frac{4\pi}{3}) \\ 1+2q\cos(\omega_m t-\frac{2\pi}{3}) & 1+2q\cos(\omega_m t-\frac{4\pi}{3}) & 1+2q\cos(\omega_m t) \\ 1+2q\cos(\omega_m t-\frac{4\pi}{3}) & 1+2q\cos(\omega_m t) & 1+2q\cos(\omega_m t-\frac{2\pi}{3}) \end{bmatrix} \tag{10}$$

$$\begin{aligned} &\text{for } \omega_m=\omega_o+\omega_i \\ &\text{and } 0\le q\le 0.5 \end{aligned}$$

CREATING PWM WAVE FORMS FOR MATRIX CONVERTER

Although, the mathematical solution for switching functions of matrix converter is well known there are limited materials to explain how switching functions are created. In this part of the study the detailed implementation of switching functions for a matrix converter will be explained.

Switching functions represented in Eq. 10 are in the form of sinusoids. However, a waveform should be in the form of 1 (i.e., switch is conducting) and 0 (i.e., switch is open), so that it can be interpreted as a switching function. Therefore, a pulse waveform should be generated which has the characteristic of the switching function in Eq. 10, i.e., it should represent the sinusoidal solution and at the same time holds the constraints in Eq. 8. A basic pattern for turning the switches on and off is shown in Fig. 2. In this pattern three equal pulses are generated in each switching period. The switching frequency ($f_{switching}$) is equal to the modulation frequency (f_m) i.e., $f_{switching} = f_m$:

$$\omega_i+\omega_o = 2\pi f_{switching} \tag{11}$$

This method is very simple to be implemented and analyzed with Fourier series (Eq. 18). However, such a square waveform not only creates the desired sinusoidal component (i.e.:

$$\frac{1}{3}(1+2q\cos(\omega_m t-\vartheta))$$

but it contains low frequency components.

Another switching option is Pulse Width Modulation (PWM). It is a flexible process which is very common in the switch mode DC/AC inverters. PWM process approximate the fundamental component while low frequency components are relatively small and other large components appear close to switching frequency (Holms, 2003).

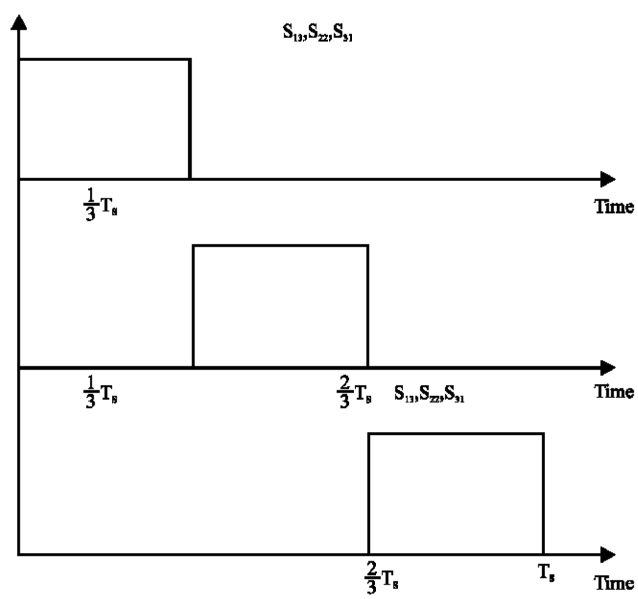

Fig. 2: A basic pattern for the switching of matrix converter

There are many solutions for analog implementation of PWM for DC/AC inverters, all of them work on the same principles (Neacsu, 2006). For matrix converter the process is a bit more complex. Three separate groups of pulses should be produced for ($S_{11}(t)$, $S_{21}(t)$ and $S_{31}(t)$). Two modulating functions $M_1(t)$ and $M_2(t)$ (with the frequency of f_m) are needed. $M_1(t)$, $M_2(t)$ and a triangular function (with the frequency of $f_{switching}$) all are applied to comparators. The first comparator gives pulses for S_{11} and provides a high output if:

$$\frac{1}{3}+\frac{2}{3}q\cos(\omega_m t) > \text{Tri}$$

and a low output if:

$$\frac{1}{3}+\frac{2}{3}q\cos(\omega_m t) < \text{Tri}$$

The second comparator gives pulses for S_{21} and provides a high output if:

$$\frac{1}{3}+\frac{2}{3}q\cos(\omega_m t) < \text{Tri} \text{ and } \text{Tri} < \frac{1}{3}+\frac{2}{3}q\cos(\omega_m t)+\frac{1}{3}+\frac{2}{3}q\cos(\omega_m t-\frac{2\pi}{3})$$

Using the fact that the sums of three switch functions in one column is equal to one, the pulses for the third switch are obtain by using logical operands: When S_{11} and S_{21} are OFF, S_{31} should be ON. This process is illustrated in Fig. 3. The simulation method is shown in Fig. 4 using MATLAB/SIMULINK.

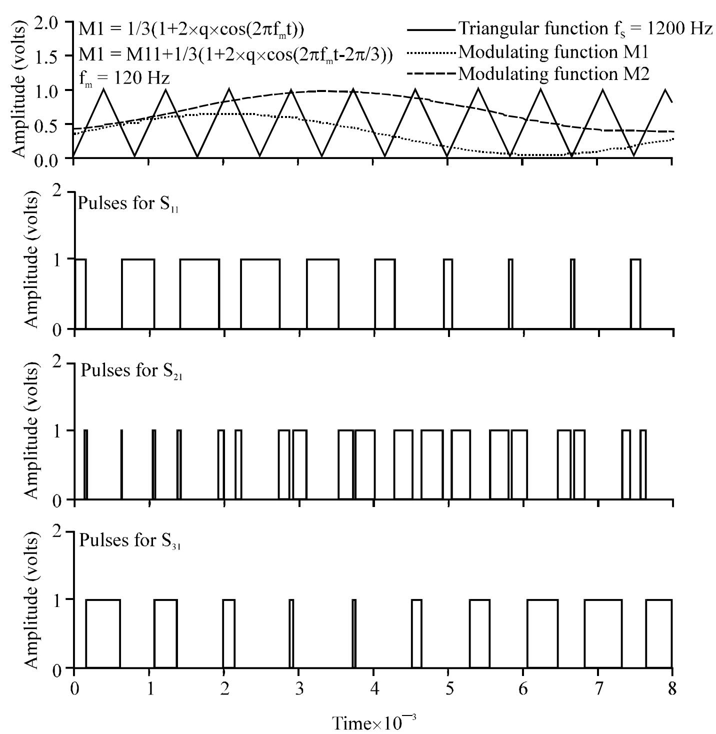

Fig. 3: PWM triangle comparison process for matrix converter (Amplitude (volts) vs. time (seconds))

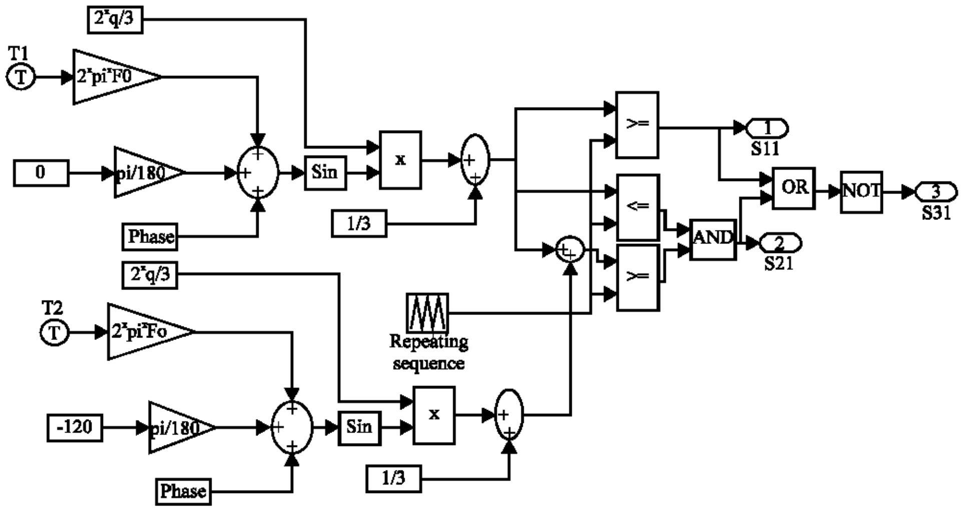

Fig. 4: Simulation method for generating PWM pulses using SIMULINK/MATLAB

Table 1: MATLAB/m-file program for generating digital PWM pulses

Source code
Function S11 = pulse(time)
S11 = 0; fm = 120; fs = 1200; q = 0.5; Ts = 1/fs;
n = floor(time/Ts);T = mod(time, Ts);
D1 = (1+*2*q*cos(2*pi*n*fm*Ts))*Ts/3;
if T<D1
S11 = 1;
else
S11 = 0;
end

Strong preferences for the modern implementation of PWM embrace Digital Signal Processors (DSP) and Field Programmed Gate Arrays (FPGA) (Neacsu, 2006; Tzou and Hsu, 1997). DSPs can be incorporated with computational software like MATLAB which enable a PC-based simulation. The formulae which were derived and fully discussed in the next section (Eq. 34-37) can be applied to a computer program. The program must create simulations of real-time calculations of the width for each pulse at the switching frequency. These individual pulses are used to generate the whole PWM signal at modulation frequency. To provide an example an m-file/MATLAB simulation program is given in Table 1.

MATHEMATICAL ANALYSIS OF SWITCHING FUNCTIONS

Fourier analysis is a powerful analytical tool for signal analysis. Fourier series is used to expand a periodic function in terms of an infinite series, while Fourier transform is the extension of Fourier series to non-periodic functions. Fourier transform gives the representations of unknown signals and is helpful to obtain the spectra of a signal in frequency domain (O'Gorman, 2000); however, in power electronics the switching functions are known and for representation of the signal in time domain the Fourier series is more applicable.

Assume a periodic function of time S(t) such that $S(t+\tau) = S(t)$. Then the periodic function can be written as an infinite sum of sinusoids known as Fourier series:

$$S(t) = \alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos(n\omega t) + b_n \sin(n\omega t) \tag{12}$$

Where:

$$\omega = \frac{2\pi}{T},\ \alpha_0 = \frac{1}{T}\int_{\tau}^{\tau+T} S(t)dt \tag{13}$$

$$\alpha_n = \frac{2}{T}\int_{\tau}^{\tau+T} S(t)\cos(n\omega t)dt \tag{14}$$

$$b_n = \frac{2}{T}\int_{\tau}^{\tau+T} S(t)\sin(n\omega t)dt \tag{15}$$

The series can also be written in the following form:

$$S(t) = \sum_{n=0}^{\infty} c_n \cos(n\omega t + \theta_n) \tag{16}$$

Where:

$$\left.\begin{array}{l} c_0 = \alpha_0;\ \theta_0 = 0;\ c_n = \sqrt{(\alpha_n + b_n)} \\ \theta_n = -\tan^{-1}\left({b_n}/{\alpha_n}\right) \end{array}\right\} \tag{17}$$

The term $c_n \cos(n\omega t+\theta_n)$ is called a harmonic of S(t). The term c_0 is the dc component and c_1 is the fundamental component. It is worth noting that the Fourier transform of any arbitrary periodic function is a sequence of impulses with weight $2\pi C_n$ located at $\omega = n\omega_0$ with $n = 0, \pm 1, \pm 2, \ldots$. Thus, the Fourier series and transform of a periodic function are closely related.

To include an example, assume a pulse train with an arbitrary period of T and equal duration of DT centered at t_0 (Fig. 5). The Fourier representation of this function would be extremely helpful in later computations. The Fourier series of this general pulse train is:

$$S_{\text{pulse train}}(t) = D + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\sin(n\pi D)}{n} \cos(n\omega t - n\omega t_0) \tag{18}$$

For PWM signals, harmonic components can be computed by Fourier analytical method developed by Black (1953) popularly known as wall method of double Fourier series. In the

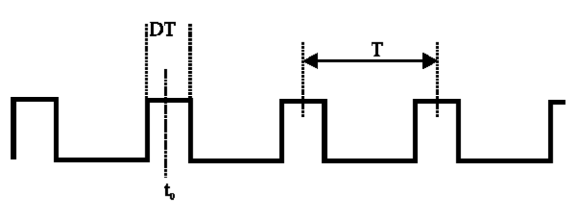

Fig. 5: Pulse train with period of T and duration of DT

following part of the paper this method is employed as a general approach to evaluate the harmonic components of PWM switching functions. For the rest we intend to introduce two other analytical approaches.

Approach 1 (general approach): Double Fourier series representation of switching functions of matrix converter: Analysis of PWM signals can be done using the double Fourier series concept. This general approach is well described and used to evaluate the harmonic content of switching functions (and output voltages) for DC/AC converters (Holms, 2003). There is a slight difference in evaluation of double Fourier series coefficients for a DC/AC inverter and matrix converter due to the difference between switching pulses. If a periodical waveform results from the modulation products of two periodical waveforms, a 3-D model and a double Fourier series is needed to find the spectrum Black (1953). The 3-D model is illustrated in Fig. 6 where the 3-D function $z = S_{11}$ (x, y) is equivalent to 1 where the line $y = \omega_m/\omega_c x$ ($\omega_m = 22\pi f_m$ and $\omega_c = 2\pi f_{switching}$) intersects the walls with vertical faces formed with modulating signals. Variables y and x are related to:

$$y = \omega_m t \qquad (19)$$

$$x = \omega_m t \qquad (20)$$

The general form of double Fourier series is (Shen *et al.*, 1997; Deslauriers *et al.*, 2005; Holms, 2003):

$$\begin{aligned} S_{11}(x, y) = \frac{1}{2}A_{00} &+ \sum_{n=1}^{\infty}(A_{0n}\cos(ny) + B_{0n}\sin(ny)) \\ &+ \sum_{m=1}^{\infty}(A_{m0}\cos(mx) + B_{m0}\sin(mx)) \\ &+ \sum_{m=1}^{\infty}\sum_{n=\pm 1}^{\pm\infty}(A_{mn}\cos(mx+ny) + B_{mn}\sin(mx+ny)) \end{aligned} \qquad (21)$$

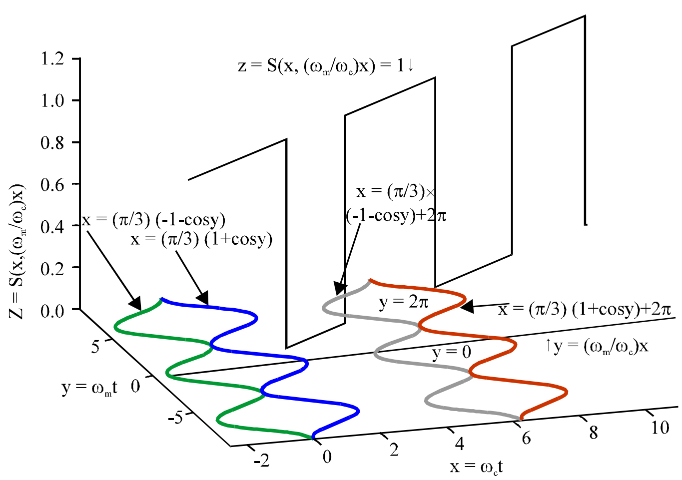

Fig. 6: 3-D model representation of PWM switching function

The Fourier coefficients are given by:

$$A_{mn} + jB_{mn} = \frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{2\pi} S_1(x,y)e^{j(mx+ny)}dxdy \tag{22}$$

With reference to Fig. 6 the value of the first switching function of matrix converter i.e., $S_{11}(x, y)$ is equal to one when $0 \le y \le 2\pi$ and:

$$-(\frac{1}{3}+\frac{1}{3}\cos y) \le \frac{x}{\pi} \le (\frac{1}{3}+\frac{1}{3}\cos y)$$

Substituting the integration limits into Eq. 22 yields to:

$$\begin{aligned}
A_{mn} + jB_{mn} &= \frac{1}{2\pi^2}\int_0^{2\pi}\int_{(-\frac{\pi}{3}-\frac{\pi}{3}\cos y)}^{(\frac{\pi}{3}+\frac{\pi}{3}\cos y)} S_1(x,y)e^{j(mx+ny)}dxdy \\
&= \frac{-j}{2\pi^2 m}\int_0^{2\pi} e^{jny}\left\{e^{j(\frac{m\pi}{3}+\frac{m\pi}{3}\cos y)} - e^{-j(\frac{m\pi}{3}+\frac{m\pi}{3}\cos y)}\right\}dy \\
&= \frac{-j}{2\pi^2 m}\left\{e^{jm\frac{\pi}{3}}\times\int_0^{2\pi} e^{jny}e^{j\frac{m\pi}{3}\cos y}dy - e^{-jm\frac{\pi}{3}}\times\int_0^{2\pi} e^{jny}e^{-j\frac{m\pi}{3}\cos y}dy\right\} \\
&= \frac{-j}{2\pi^2 m}\left\{e^{jm\frac{\pi}{3}}\times 2\pi j^n\times J_n(\frac{m\pi}{3}) - e^{-jm\frac{\pi}{3}}\times 2\pi j^n\times J_n(\frac{-m\pi}{3})\right\} \\
&= \frac{-j}{2\pi^2 m}\times 2\pi\left\{e^{jm\frac{\pi}{3}}\times j^n\times J_n(\frac{m\pi}{3}) - e^{-jm\frac{\pi}{3}}\times j^n\times(-1)^n\times J_n(\frac{m\pi}{3})\right\} \\
&= \frac{-j}{2\pi^2 m}\times 2\pi\times J_n(\frac{m\pi}{3})\left\{e^{j\pi(\frac{m}{3}+\frac{n}{2})} - e^{-j\pi(\frac{m}{3}+\frac{n}{2})}\right\} \\
&= \frac{2}{\pi m}\times J_n(\frac{m\pi}{3})\times\sin(\pi(\frac{m}{3}+\frac{n}{2}))
\end{aligned} \tag{23}$$

Note that:

$$\int_0^{2\pi} e^{jm\theta\cos y}e^{jny}dy = 2\pi j^n J_n(m\theta)$$

and $J_n(-m\theta) = (-1)^n j_n(m\theta)$, where, $j_n(x)$ is the nth order of Bessel function.

For n=0:

$$B_{m0} = 0; A_{m0} = \frac{2}{\pi m}\times J_0(\frac{m\pi}{3})\times\sin(\frac{\pi m}{3})$$

For m=0:

$$A_{0n} + jB_{0n} = \frac{1}{2\pi^2}\int_0^{2\pi}\int_{(-\frac{\pi}{3}-\frac{\pi}{3}\cos y)}^{(\frac{\pi}{3}+\frac{\pi}{3}\cos y)} S_1(x,y)e^{jny}dxdy = \frac{1}{2\pi^2}\times 2\int_0^{2\pi}(\frac{\pi}{3}+\frac{\pi}{3}\cos y)e^{jny}dy = \frac{1}{\pi^2}\left(\underbrace{\int_0^{2\pi}\frac{\pi}{3}e^{jny}dy}_{\text{zero}} + \underbrace{\int_0^{2\pi}\frac{\pi}{3}\cos y e^{jny}dy}_{\text{zero for all n}\neq 1}\right) = \begin{cases} 0 & n \neq 1 \\ \frac{1}{3} & n = 1 \end{cases} \Rightarrow A_{01} = \frac{1}{3}$$

For m = n = 0:

$$A_{00} = \frac{1}{2\pi^2}\int_0^{2\pi}\int_{(-\frac{\pi}{3}-\frac{\pi}{3}\cos y)}^{(\frac{\pi}{3}+\frac{\pi}{3}\cos y)} dxdy = \frac{1}{2\pi^2}\times 2\int_0^{2\pi}(\frac{\pi}{3}+\frac{\pi}{3}\cos y)dy = \frac{1}{\pi^2}\left(\int_0^{2\pi}\frac{\pi}{3}dy + \underbrace{\int_0^{2\pi}\frac{\pi}{3}\cos y dy}_{\text{zero}}\right) = \frac{2}{3} \Rightarrow \frac{1}{2}A_{00=}\frac{1}{3}$$

Hence, the Fourier series of $S_{11}(t)$ can be expressed:

$$S_{11}(t) = \frac{1}{3} + \frac{1}{3}\cos(\omega_m t) + \sum_{m=1}^{\infty}\frac{2}{\pi m}J_0(\frac{m\pi}{3})\sin(\frac{m\pi}{3})\times\cos(m\omega_c t) + \sum_{m=1}^{\infty}\sum_{n=\pm 1}^{\pm\infty}\frac{2}{\pi m}J_n(\frac{m\pi}{3})\sin(\pi(\frac{m}{3}+\frac{n}{2}))\times\cos(m\omega_c t + n\omega_m t) \quad (24)$$

More often electrical engineers are interested in harmonics of cos ($\omega_m t$), thus:

$$\overset{\frac{\omega_c}{\omega_m}=p}{\Rightarrow} \quad (25)$$

$$S_{11}(t) = \frac{1}{3} + \frac{1}{3}\cos(\omega_m t) + \sum_{m=1}^{\infty}\frac{2}{\pi m}J_0(\frac{m\pi}{3})\sin(\frac{m\pi}{3})\times\cos(mp\omega_m t) + \sum_{m=1}^{\infty}\sum_{n=\pm 1}^{\pm\infty}\frac{2}{\pi m}J_n(\frac{m\pi}{3})\sin(\pi(\frac{m}{3}+\frac{n}{2}))\times\cos((mp+n)\omega_m t)$$

Observation of this expression provides useful and relevant information about amplitude of the components at the modulating frequency. First, it would normally be expected that the DC component and the fundamental component, be equal to 1/3 for $q = 0.5$. The third term of Eq. 25 gives a portion of the amplitude of harmonics which are equal or higher than the switching frequency. Because of the presence of $\sin(m\pi/3)$, for all m multiples of 3 the third term is equal to zero. The fourth term also gives a portion of the amplitude of the harmonics which are equal or higher than the switching frequency and also it gives the amplitude of other harmonics. In order to find a desired harmonic e.g., the second harmonic the equation $mp+n = 2$ should be solved for all positive integer values of m and negative and positive integer values of n. Note that for the harmonics equal or higher than the switching frequency two equations should be taken into consideration, e.g., for the tenth harmonic:

$$\begin{cases} mp + n = 10 \\ mp = 10 \end{cases}$$

should separately be solved for all positive integer values of m and negative and positive integer values of 2.

Approach 2: Representation of switching functions of matrix converter using Fourier series of a pulse train with variable duration: In this approach it is assumed that the switching function $S_{11}(t)$, generated by PWM process, is the same pulse train as represented in Fig. 5. However, the duty ratio (i.e., D) of the pulse train illustrated in Fig. 5 is fixed and has a constant value but there is no particular value of D for the PWM signal and it varies with time:

$$D_{11}(t) = \frac{1}{3} + \frac{1}{3}\cos(\omega_m t) \ \text{(for } q = 0.5) \quad (26)$$

Substituting this time variable function into Fourier series Eq. 18 results in:

$$S_{11}(t) = \frac{1}{3} + \frac{1}{3}\cos(\omega_m t) + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi(\frac{1}{3} + \frac{1}{3}\cos(\omega_m t)))}{n}\cos(n\omega_s t) \tag{27}$$

It is no longer a Fourier series, owing to the appearance of the term:

$$\sin(n\pi(\frac{1}{3} + \frac{\cos(\omega_m t)}{3}))$$

This expression can be analyzed using following equations and again it can be analyzed by taking advantages of Bessel functions.

The term:

$$\sin(n\pi(\frac{1}{3} + \frac{\cos(\omega_m t)}{3}))$$

can be calculated by the following equations:

$$\sin(\frac{n\pi}{3} + \frac{n\pi\cos(\omega_m t)}{3}) = \sin(\frac{n\pi}{3})\cos(\frac{n\pi\cos(\omega_m t)}{3}) + \cos(\frac{n\pi}{3})\sin(\frac{n\pi\cos(\omega_m t)}{3}) \tag{28}$$

$$\sin(\frac{n\pi\cos(\omega_m t)}{3}) = 2J_1(\frac{n\pi}{3})\cos(\omega_m t) - 2J_3(\frac{n\pi}{3})\cos(3\omega_m t) + 2J_5(\frac{n\pi}{3})\cos(5\omega_m t) - \ldots \tag{29}$$

$$\cos(\frac{n\pi\cos(\omega_m t)}{3}) = J_0(\frac{n\pi}{3}) - 2J_2(\frac{n\pi}{3})\cos(2\omega_m t) + 2J_4(\frac{n\pi}{3})\cos(4\omega_m t) - 2J_6(\frac{n\pi}{3})\cos(6\omega_m t) + \ldots \tag{30}$$

where, $J_n(x)$ is the nth order of Bessel function.

Development of the series in Eq. 27 yields to:

$$\begin{aligned}
H &= \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(\frac{n\pi}{3} + \frac{n\pi}{3}\times\cos(\omega_m t))}{n}\cos(n\omega_s t) \\
&= \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(\frac{n\pi}{3})\cos(\frac{n\pi}{3}\cos(\omega_m t)) + \cos(\frac{n\pi}{3})\sin(\frac{n\pi}{3}\cos(\omega_m t))}{n}\cos(n\omega_s t) \\
&= \frac{2}{\pi}\left\{\sum_{n=1}^{\infty}\frac{\sin(\frac{n\pi}{3})}{n}\cos(\frac{n\pi}{3}\cos(\omega_m t)) + \sum_{n=1}^{\infty}\frac{\cos(\frac{n\pi}{3})}{n}\sin(\frac{n\pi}{3}\cos(\omega_m t))\right\}\cos(n\omega_s t) \\
&= \frac{2}{\pi}\left\{\sum_{n=1}^{\infty}\frac{\sin(\frac{n\pi}{3})}{n}(J_0(\frac{n\pi}{3}) - 2J_2(\frac{n\pi}{3})\cos(2\omega_m t) + 2J_4(\frac{n\pi}{3})\cos(4\omega_m t) - \ldots)\right\} + \sum_{n=1}^{\infty}\frac{\cos(\frac{n\pi}{3})}{n}(2J_1(\frac{n\pi}{3})\cos(\omega_m t) - 2J_3(\frac{n\pi}{3})\cos(3\omega_m t) + 2J_5(\frac{n\pi}{3})\cos(5\omega_m t) - \ldots)\Bigg\}\cos(n\omega_s t) \\
&= \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left\{\sin(\frac{n\pi}{3})\left[J_0(\frac{n\pi}{3}) + \sum_{m=1}^{\infty}(-1)^m\times 2J_{2m}(\frac{n\pi}{3})\cos(2m\omega_m t)\right] + s\cos(\frac{n\pi}{3})\left[\sum_{m=1}^{\infty}(-1)^{m-1}\times 2J_{2m-1}(\frac{n\pi}{3})\cos((2m-1)\omega_m t\right]\right\}\cos(n\omega_s t) \\
&= \frac{2}{\pi}\left\{\sum_{n=1}^{\infty}\frac{\sin(\frac{n\pi}{3})}{n}J_0(\frac{n\pi}{3})\cos(n\omega_s t) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\sin(\frac{n\pi}{3})}{n}(-1)^m\times 2J_{2m}(\frac{n\pi}{3})\cos(2m\omega_m t)\cos(n\omega_s t) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\cos(\frac{n\pi}{3})}{n}(-1)^{m-1}\times 2J_{2m-1}(\frac{n\pi}{3})\cos((2m-1)\omega_m t)\cos(n\omega_s t)\right\}
\end{aligned} \tag{31}$$

The closed form solution is:

$$\begin{aligned} S_{11}(t) &= \frac{1}{3} + \frac{1}{3}\cos(\omega_m t) \\ &+ \frac{2}{\pi}\left\{ \sum_{n=1}^{\infty} \frac{\sin(\frac{n\pi}{3})}{n} J_0(\frac{n\pi}{3})\cos(n\omega_s t) \right. \\ &+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\sin(\frac{n\pi}{3})}{n} (-1)^m \times 2J_{2m}(\frac{n\pi}{3})\cos(2m\omega_m t)\cos(n\omega_s t) \\ &+ \left. \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\cos(\frac{n\pi}{3})}{n} (-1)^{m-1} \times 2J_{2m-1}(\frac{n\pi}{3})\cos((2m-1)\omega_m t)\cos(n\omega_s t) \right\} \end{aligned} \tag{32}$$

This expression verifies that the desired components appear, i.e. the DC component and the fundamental component are equal to what was expected. The third term of Eq. 32 is exactly the third term of Eq. 25 and gives a portion of the amplitude of harmonics equal and higher than the switching frequency. In the fourth and the fifth terms we observe the product of cosine functions. Since we are concerned with modulation frequency using the product identities yields:

$$\begin{aligned} &\frac{\omega_c}{\omega_m} = p \\ &\Rightarrow \\ &\cos(2m\omega_m t)\cos(n\omega_s t) = \frac{1}{2}\left(\cos((2m+pn)\omega_m t) + \cos((2m-pn)\omega_m t)\right) \\ &\cos((2m-1)\omega_m t)\cos(n\omega_s t) = \frac{1}{2}\left(\cos((2m-1+pn)\omega_m t) + \cos((2m-1-pn)\omega_m t)\right) \end{aligned} \tag{33}$$

In order to find a specific harmonic e.g., the second harmonic four equations (2m+pn = 2, 2m-pn = 2, 2m-1+pn = 2 and 2m-1-pn = 2) should separately be solved for all positive integer values of m and n.

Approach 3: Simple representation of switching functions of matrix converter using the sum of one dimensional Fourier series: Owing to the difficulty in the evaluation of Bessel function and double Fourier coefficients, another method will be introduced to analyze the PWM signals.

Let define $p = \frac{f_{switching}}{f_m}$. Then, there are p pulses in each $T_m = \frac{1}{f_m}$. If the width of each pulse in each sampling time $T_s = \frac{1}{f_{switching}}$ was known, it would be easy to use a simple sum or the superposition of the p pulses to generate the whole PWM signal. However, the calculation of pulses' width is actually a tedious work because a nonlinear equation should be solve to find the crossing points of the carrier and modulating function in each T_s. Fortunately an assumption would help in this case. Since, in the PWM process the frequencies are chosen so, that $f_{swiching} >> f_m$, then: $T_s << T_m$. During the sampling time (T_s) the variable t in Eq. 10 can be assumed constant. It would result in a simple calculation for pulses width. For example for $f_i = f_o = 60$ Hz, the modulation frequency is $f_m = f_i + f_o = 120$ Hz. For switching (or sampling) frequency of $f_s = 1200$ Hz one pulse is produced in each T_s and ten pulses are produced in each T_m. Let us define the width of pulses for S_{11} as D_1. The width of the pulses can be calculated with the following equation:

$$\left.\begin{array}{l} D_1^n = \frac{1}{3}(1+2q\cos(\omega_m \frac{n}{f_s})) \\ n = 1:(\frac{f_s}{f_m}) \end{array}\right\} \quad (34)$$

The above index (i.e., n) represents the nth pulse and index 1 is used for S_{11}. For S_{21} and S_{31} using Eq. 35 and Eq. 36 would give the pulses' width:

$$D_2^n = (1+2q\cos(\omega_m \frac{n}{f_s} - \frac{2\pi}{3})) \times \frac{1}{3} \quad (35)$$

$$D_3^n = (1+2q\cos(\omega_m \frac{n}{f_s} - \frac{4\pi}{3})) \times \frac{1}{3} \quad (36)$$

Again if at any sampling time, T_s, the time is assumed constant, then, for each p pulse, t_a^n, the center of the pulse, can be computed by:

$$\left.\begin{array}{l} \text{for} S_{11}(t): t_1^n = \frac{D_1^n}{2} + \frac{(n-1)}{f_s} \\ \text{for} S_{21}(t): t_2^n = D_1^n + \frac{D_2^n}{2} + \frac{(n-1)}{f_s} \\ \text{for} S_{31}(t): t_3^n = D_1^n + D_2^n + \frac{D_3^n}{2} + \frac{(n-1)}{f_s} \\ n = 1:p \end{array}\right\} \quad (37)$$

In Fig. 7 for the ratio of switching frequency (f_s = 1200 Hz) to modulating frequency (f_m = 120 Hz), equal to ten ($p = f_s/f_m = 10$), the pulses are delineated in each T_m. The Fourier series of each pulse in the function $S_{11}(t)$ can be evaluated from Eq. 18. Let $S^p_{11}(t)$ denote the Fourier series of each pulse in $S_{11}(t)$. Substituting the duration of D_1^p centered at t_1^p in Fourier series of the pulse train yields:

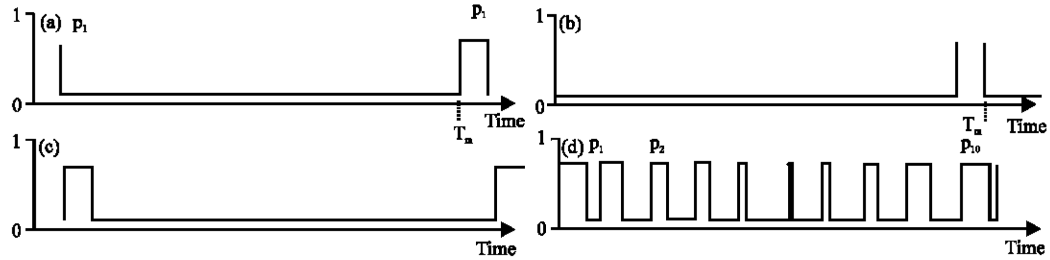

Fig. 7(a-d): S_{11} can be assumed as a combination of p individual pulse trains, (a) 1st, (b) 2nd, (c) 10th and (d) Sum of 10 pulse trains

$$S^{p}_{11}(t) = D_1^{p}(t) + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi D_1^{p}(t))}{n}\cos(n\omega_s t - n\omega t_1^{p}) \quad (38)$$

$S_{11}(t)$ can be assumed as a combination of individual pulse trains (Fig. 7). If the Fourier series of each pulse is defined as S_{11}^{p} then the function $S_{11}(t)$ is simply computed by:

$$S_{11}(t) = \sum_{i=1}^{p} S_{11}^{i} \quad (39)$$

The same formulation can be derived for $S_{ij}(t)$ (i,j = 1,2,3). This would be an easier approach comparing to Eq. 25 and Eq. 32 for understanding the properties of PWM and computing harmonic component of the function $S_{ij}(t)$. For example if the fundamental harmonic of $S_{11}(t)$ is important to be known, it can simply be calculated with the sum of p phasors and there would be no evaluation of Bessel function or complex double Fourier series. Referring to Eq. 38 the phasor of fundamental component for the first pulse (i.e., p = 1) is:

$$\left|C_1^{1}\right| = \frac{2}{\pi}\sin(\pi D_1^{1}), \prec C_1^{1} = \theta_1^{1} = \omega_s t_1^{1}$$

Simply the phasors of fundamental component for nine other pulses in $S_{11}(t)$ can be evaluated and so, the fundamental harmonic of $S_{11}(t)$ can be expressed:

$$c_1 < \theta_1 = c_1^{1} < \theta_1^{1} + c_1^{2} < \theta_1^{2} + \ldots + c_1^{p} < \theta_1^{p} \quad (40)$$

A complete result of such calculation is given in the computation next section.

COMPUTATION RESULTS

In this part, the calculation results of harmonic analysis for switching function of matrix converter based on the three approaches are presented. Matrix converter is switched at the frequency of f_s = 1200 Hz and the input frequency is assumed to be f_i = 60 Hz. The desired output frequency is also f_o = 60 Hz. Referring to Eq. 9, $f_m = f_o - f_i = 0$ is a valid choice but it eliminates opportunities for control. In this case the choice for modulating frequency is $f_m = f_i + f_o$ = 120 Hz. Figure 8 represents the amplitudes of harmonics evaluated by MATLAB based on Eq. 25. Harmonics with magnitude of less than 0.02 are not shown in Fig. 8. It can be inferred that the significant components are DC, fundamental, tenth and also upper and lower sidebands of the tenth harmonic (switching frequency). Results for magnitude of the DC component and harmonics (one up to ten) are given in Table 2. Since, the double Fourier series approach is considered to be the general method, the results of calculations, based on two other approaches can be compared with that. Comparison of calculation results of the simple approach with the general approach provides verification of accuracy for this approach. Approach 3 is also applied to three switching pulse functions S_{11}, S_{21}, S_{31} and the DC components, fundamental components and other harmonics are evaluated by MATLAB. Both amplitude and phase angle of sinusoidal functions are represented in Table 3. Results show that the switching functions have the desired components of Eq. 10. The comparative information that can be gained from the inspection of Table 3 is worth

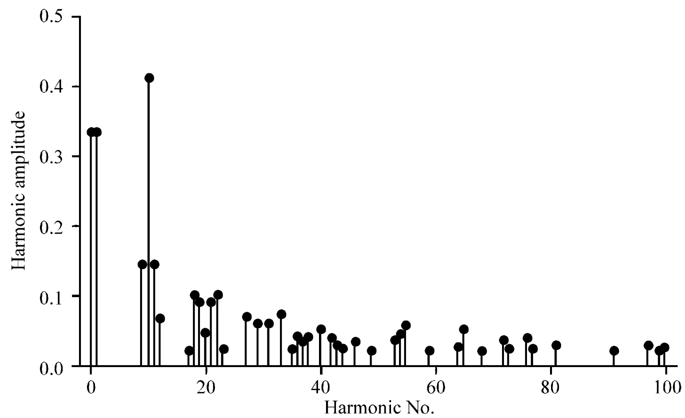

Fig. 8: Harmonics of S_{11}

Table 2: Calculation results for magnitude of switching function S_{11}

Harmonic number	Approach 1	Approach 2	Approach 3
DC component (Volts)	0.333333333	0.333333333	0.333333333
Fundamental (120 Hz)	0.333333335	0.333333338	0.331057656
Second harmonic (240 Hz)	0.000000074	0.000000149	0.000498162
Third harmonic (360 Hz)	0.000000658	0.000001316	0.000133992
Fourth harmonic (480 Hz)	0.000015171	0.000030342	0.000071747
Fifth harmonic (600 Hz)	0.000099713	0.000199426	0.000000000
Sixth harmonic (720 Hz)	0.001634071	0.003268143	0.000297166
Seventh harmonic (840 Hz)	0.007107575	0.014215151	0.005690267
Eighth harmonic (960 Hz)	0.068900939	0.137801882	0.049949365
Ninth harmonic (1080 Hz)	0.144840748	0.289681496	0.175368518
Tenth harmonic (1200 Hz)	0.410228268	0.410228595	0.409573399

Table 3: Harmonic calculations (magnitude (volts) and phase angle degrees) based on approach-3 for three Switching functions S_{11}, S_{21} and S_{31}

Harmonic number	S_{11}	S_{22}	S_{33}
DC (Volts)	0.3333	0.3333	0.3333
120 Hz (Fnd)	0.3311<91.2	0.3371<211.8	0.3311<-28.6
240 Hz (2nd)	0.0005<-41.6	0.0002<63.6	0.0005<168.8
360 Hz (3rd)	0.0001<97.2	0.0001<95.4	0.0001<93.5
480 Hz (4th)	0.0001<-38.0	0.0001<-52.5	0.0001<-67.4
600 Hz (5th)	0.0000<-79.5	0.0000<-12.0	0.0000<180.7
720 Hz (6th)	0.0003<-23.9	0.0003<250.7	0.0003<165.2
840 Hz (7th)	0.0057<-71.3	0.0024<-77.4	0.0057<-83.4
960 Hz (8th)	0.0499<-4.7	0.0626<134.4	0.0499<-86.4
1080 Hz (9th)	0.1754<-78.5	0.1328<166.2	0.1754<50.9
1200 Hz (10th)	0.4096<8.1	0.4063<18.0	0.4096<27.8
1320 (11th)	0.1150<104.9	0.1579<229.8	0.1150<-5.3
1440 (12th)	0.0876<4.0	0.0744<261.5	0.0876<159.1
1560 (13th)	0.0079<149.6	0.0158<113.3	0.0079<77.13

noting. For example as it was expected that fundamental components of three switching functions have 120° phase difference but the same amplitudes. The undesired components (i.e., low frequencies) are low enough that a low-pass filter would reduce distortion. The results obtained from approach-3 show that this method is a good approximation to the PWM harmonic content.

CONCLUSION

In this study, three analytical approaches for the study of the harmonic spectra of PWM switching functions of a three-phase matrix converter were presented. Three different ways were also explored for generating switching pulses of a three-phase matrix converter. Fully detailed procedure of creating switching pulses was demonstrated. The first method was a technique with equal pulses in each switching period. In the second method triangular carrier was employed in creating PWM pulses. The third method was a digital based procedure for creating switching pulses. In this method the width of each pulse, in each switching period, was computed and then the whole switching pulses were generated. Exact mathematical expressions were derived for the spectra of the switching functions. In order to cancel the requirements for computing the double Fourier series coefficients and Bessel functions, a simple approach was proposed to analyze the switching function based on one dimensional Fourier series of a pulse train. Calculations of harmonic components of switching functions were executed on the basis of the proposed analytical technique and compared with other analytical approaches.

REFERENCES

Alesina, A. and M. Venturini, 1981. Solid-state power conversion: A Fourier analysis approach to generalized transformer synthesis. IEEE Trans. Circuits Syst., 28: 319-330.

Black, H.S., 1953. Modulation Theory. Van Nostrand Ltd., Co., New York pages: 363.

Casadei, D., G. Serra and A. Tani, 1998. A general approach for the analysis of the input power quality in matrix converters. IEEE Trans. Power Electron., 13: 882-891.

Deslauriers, I., N. Avdiu and B.T. Ooi, 2005. Naturally sampled triangle carrier PWM bandwidth limit and output spectrum. IEEE Trans. Power Electron., 20: 100-106.

Holms, G.D., 2003. Pulse Width Modulation for Power Converters. John Wiley and Sons, USA.

Kim, S., Y.D. Yoon and S.K. Sul, 2010. Pulse width modulation method of matrix converter for reducing output current ripple. IEEE Trans. Power Electron., 25: 2620-2629.

Lou, F.L. and Z.Y. Pan, 2006. Sub-envelope modulation method to reduce total harmonic distortion of AC/AC matrix converters. IEEE Proc. Elect. Power Appl., 153: 856-863.

Marouchos, C., 2006. The Switching Function Analysis of Power Electronic Circuits. 1st Edn., IET., London, ISBN: 9780863413513, pages: 297.

Marouchos, C., M.K. Darwish and M. El-Habrouk, 2010. New mathematical model for analyzing three-phase controlled rectifier using switching functions. IET Power Electron., 3: 95-110.

Mirkazemi-Moud, M., B.W. Williams and T.C. Green, 1994. A novel simulation technique for the analysis of digital asynchronous pulse width modulation. IEEE Trans. Industry Applicat., 30: 1284-1289.

Neacsu, D.O., 2006. Power-Switching Converters: Medium and High Power. CRC/Taylor and Francis, London, ISBN: 9780824726256, pages: 365.

O'Gorman, T., 2000. Discrete Fourier transform harmonic analysis of digitally-generated PWM waveforms which are distorted by switch dead time. Proceedings of the 35th IEEE Industry Applications Society Annual Meeting, October 2000, Rome, Italy, pp: 2197-2204.

Shen, J., J.A. Taufiq and A.D. Mansell, 1997. Analytical solution to harmonic characteristics of traction PWM converters. IEE Pro-Electr. Power Appl., 144: 158-168.

Tzou, Y.Y. and H.J. Hsu, 1997. FPGA realization of space-vector PWM control IC for three-phase inverters. IEEE Trans. Power Electron., 12: 953-963.

Wheeler, P.W., J. Rodriguez, J.C. Clare, L. Empringham and A. Weinstein, 2002. Matrix converters: A technology review. IEEE Trans. Ind. Electron., 49: 276-288.