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Bayesian Using Extension of Jeffreys Estimator of Weibull Distribution Based on Type-I and II Cnsored Data

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ABSTRACT

Weibull distribution has drawn a lot of attention from statisticians working on both theory and methods and even other fields of applied statistics. In this study, the performance of Maximum Likelihood Estimator and the Bayesian Estimator using Jeffreys prior and extension of Jeffreys prior information for estimating the scale parameter, survival function and the hazard function of the Weibull distribution given shape with type-I and II censored data was estimated. The comparisons are made with respect to the Mean Square Error (MSE) for various sample sizes with different values of the scale and shape parameters, extension of Jeffreys prior and different percentage of censoring. The Bayesian using extension of Jeffreys prior based on type-I and II is better estimator compared to the other estimations.

Key words: Weibull distribution, Bayesian method, type-I and II censored data, survival and hazard functions

INTRODUCTION

The Weibull distribution has the widest variety of many applications in different areas, including lifetime testing, reliability theory and others. Sinha (1986) estimated the scale and shape parameters of Weibull distribution by MLE and Bayesian methods. Smith and Naylor (1987) developed the maximum likelihood and Bayesian approach and compared them using the Weibull distribution. Sun (1997) estimated the two parameters of Weibull distribution where he compared Jeffreys prior with that of the reference prior under Bayesian methods. Hossain and Zimmer (2003) estimated the parameters of Weibull distribution using complete and censored samples by MLE and least squares method. Singh *et al.* (2005) obtained Bayesian and MLE for Exponentiated Weibull distribution based type-II censoring. Soliman *et al.* (2006) estimated Weibull distribution by using MLE and Bayesian estimation following by estimated the hazard and reliability functions. Kantar and Senoglu (2008) reported their findings on the comparative study for the location and scale parameters of the Weibull distribution with a given shape parameter. Kundu and Howlader (2010) obtained Bayesian inference and prediction of the inverse Weibull distribution based type-II censored data. Pandey *et al.* (2011) compared Bayesian estimation and MLE of the scale parameter in Weibull distribution with known shape.

The objective of this study is to estimate the scale parameter, survival and hazard functions of the Weibull distribution based on type-I and II censored data by using Bayesian approach and compared to MLE using Mean Square Error (MSE) to determine the better estimator under several conditions.

MATERIALS AND METHODS

Maximum likelihood estimation of Weibull censored data: Concept of maximum likelihood estimation on Weibull distribution with censored data was introduced here. Let $(x_1,\dots, x_n)$ be the set of n random lifetime from Weibull distribution with parameters λ and α.

The probability density function of Weibull distribution is:

$$f(x;\lambda,\alpha)=\frac{\alpha}{\lambda}x^{\alpha-1}\exp\left(-\frac{x^{\alpha}}{\lambda}\right)$$

where, the Cumulative Distribution Function (CDF) of the Weibull distribution is given as:

$$F(x;\lambda,\ \alpha)=1-\exp\left(-\frac{x^{\alpha}}{\lambda}\right)$$

with λ as the scale parameter and α the shape parameter of the Weibull distribution.

- **Maximum likelihood estimation based on type-I censored data:** The likelihood function as in the study of Klein and Moeschberger (2003) is:

$$L(\lambda,\ \alpha\,|\,x)=\prod_{i=1}^{n}[f(x_i;\lambda,\ \alpha)]^{\delta_i}\ [S(x_i;\lambda,\ \alpha)]^{1-\delta_i} \tag{1}$$

where, δ_i is 1 for failure and δ_i is 0 for censored observation and S(.) is the survival function.

The logarithm of the likelihood function can be expressed as follows:

$$\ln L(\lambda\,|\,\alpha,\ t)=\sum_{i=1}^{n}\left[\delta_i\left(\ln\alpha-\ln\lambda+(\alpha-1)\ln x_i\right)-\frac{x_i^{\alpha}}{\lambda}\right] \tag{2}$$

To obtain the equations for the unknown parameters, we differentiate Eq. 2 partially with respect to the parameters λ and α and equal it to zero. The resulting equations are given as follows:

$$\frac{\partial L(\theta\,|\,\alpha,\ x)}{\partial\lambda_i}=-\frac{\sum_{i=1}^{n}\delta_i}{\lambda}+\frac{\sum_{i=1}^{n}x_i^{\alpha}}{\lambda^2} \tag{3}$$

$$\frac{\partial L(\lambda\,|\,\alpha,x)}{\partial\alpha_i}=\frac{\sum_{i=1}^{n}\delta_i}{\alpha}+\sum_{i=1}^{n}\delta_i\ln(x_i)-\frac{\sum_{i=1}^{n}x_i^{\alpha}\ln(x_i)}{\lambda} \tag{4}$$

Let Eq. 3 equals to zero, then the maximum likelihood estimator for the scale parameter of Weibull distribution is:

$$\hat{\lambda}_M = \frac{\sum_{i=1}^{n} x_i^{\alpha}}{\sum_{i=1}^{n} \delta_i} \tag{5}$$

The shape parameter cannot be solved analytically and for that we use the Newton Raphson method to find the numerical solution following (Hossain and Zimmer, 2003).

The estimate of the survival function for Maximum Likelihood Estimation of Weibull distribution is:

$$\hat{S}_M(t) = \exp\left(-\frac{t^{\alpha}}{\hat{\lambda}_M}\right) = \exp\left(-\frac{t^{\alpha}\sum_{i=1}^{n}\delta_i}{\sum_{i=1}^{n} x_i^{\alpha}}\right) \tag{6}$$

The estimate of the hazard function of Weibull distribution given shape by Maximum Likelihood Estimation is as follows:

$$\hat{h}_M(t) = \frac{f(t)}{S(t)} = \frac{\alpha t^{\alpha-1}}{\sum_{i=1}^{n} x_i^{\alpha}}\left(\sum_{i=1}^{n}\delta_i\right) \tag{7}$$

- **Maximum likelihood estimation based on type-II censored data:** The likelihood function is:

$$L(\lambda,\alpha \mid x) = \frac{n!}{(n-r)!}\left[\prod_{i=1}^{r}[f(x_i;\lambda,\alpha)]\right][1-F(x_r;\lambda,\alpha)]^{n-r} = \frac{n!}{(n-r)!}\left[\prod_{i=1}^{r}\frac{\alpha}{\lambda}x_i^{\alpha-1}\exp\left(-\frac{x_i^{\alpha}}{\lambda}\right)\right]\left[\exp\left(-\frac{x_r^{\alpha}}{\lambda}\right)\right]^{n-r} \tag{8}$$

The logarithm of the likelihood function can be expressed as follows:

$$\ln L(\lambda,\alpha \mid x) = \log\frac{n!}{(n-r)!} + r\log(\alpha/\lambda) + (\alpha-1)\sum_{i=1}^{r}\log(x_i) - \frac{\sum_{i=1}^{r} x_i^{\alpha}}{\lambda} + (n-r)\left(-\frac{x_r^{\alpha}}{\lambda}\right) \tag{9}$$

To obtain the equations for the unknown parameters, Eq. 9 is differentiated partially with respect to the parameters λ and α and equal it to zero. The resulting equations are given, respectively as:

$$\frac{\partial L(\lambda \mid \alpha,x)}{\partial\lambda} = -\frac{r}{\lambda} + \frac{\sum_{i=1}^{r} x_i^{\alpha}}{\lambda^2} + \frac{(n-r)x_r^{\alpha}}{\lambda^2} \tag{10}$$

$$\frac{\partial L(\alpha \mid \lambda,x)}{\partial\alpha} = \frac{r}{\alpha} + r\log(x) - \frac{\sum_{i=1}^{r} x_i^{\alpha}\log(x_i)}{\lambda} - \frac{(n-r)x_r^{\alpha}\log(x_r)}{\lambda} \tag{11}$$

Let Eq. 10 equals to zero, then the maximum likelihood estimator for the scale parameter of Weibull distribution is:

$$\hat{\lambda} = \frac{\sum_{i=1}^{r} x_i^{\alpha} + (n-r)x_r^{\alpha}}{r} \tag{12}$$

The shape parameter cannot be solved analytically and for that we employed Newton Raphson method.

The estimate of the survival function for MLE based type-II censored data:

$$\hat{S}_M(t) = \exp\left(-\frac{t^{\alpha}}{\hat{\lambda}_M}\right) = \exp\left(-\frac{r\,t^{p}}{\sum_{i=1}^{r} x_i^{\alpha} + (n-r)x_r^{\alpha}}\right) \tag{13}$$

The estimate of the hazard function of Weibull based type-II censored data:

$$\hat{h}_M(t) = \frac{f(t)}{S(t)} = = \frac{r\alpha}{\sum_{i=1}^{r} x_i^{\alpha} + (n-r)x_r^{\alpha}} t^{\alpha-1} \tag{14}$$

Bayesian estimation of weibull based on type-I censored data: The Bayesian estimator with Jeffreys prior is described here, where Jeffreys prior is the square root of the determinant of the Fisher information matrix:

$$I(\lambda) = -E\left(\frac{\partial^2 \log L(\lambda)}{\partial \lambda^2}\right)$$

Then the Jeffreys prior is:

$$g_1(\lambda) = k\frac{1}{\lambda} \tag{15}$$

where, k is a constant.

The posterior probability density function of λ given the data $(t_1, \ldots, t_n)$ is obtained by dividing the joint probability density function with the marginal density function (Ahmed *et al*., 2012):

$$\prod\nolimits_{BJ1}(\theta \mid p, t) = \frac{\left(\sum_{i=1}^{n} t_i^{p}\right)^{\sum_{i=1}^{n}\delta_i}}{\theta^{\sum_{i=1}^{n}\delta_i + 1}\left(\sum_{i=1}^{n}\delta_i - 1\right)!} \exp\left(-\frac{\sum_{i=1}^{n} t_i^{p}}{\theta}\right) \tag{16}$$

With this, the Bayesian estimates for the scale parameter of Weibull distribution under squared error loss function are given as:

$$\hat{\theta}_{BJ1} = \int_0^\infty A(\theta-\hat{\theta})^2 \prod_{BJ1}(\theta \mid p,t)\, d\theta = \frac{\sum_{i=1}^{n} t_i^p}{\sum_{i=1}^{n}\delta_i - 1} \tag{17}$$

The estimator for survival function of Weibull distribution given shape by Bayesian using Jeffreys prior obtained as:

$$\hat{S}_{BJ1}(t) = \int_0^\infty \exp\left(-\frac{t^p}{\theta}\right) \prod_{BJ1}(\theta \mid p,t)\, d\theta = \left(1 + \frac{t^p}{\sum_{i=1}^{n} t_i^p}\right)^{-\sum_{i=1}^{n}\delta_i} \tag{18}$$

The estimator for hazard function of Weibull distribution given shape by Bayesian using Jeffreys prior obtained as:

$$\hat{h}_{BJ1}(t) = \int_0^\infty \frac{p}{\theta} t^{p-1} \prod_{BJ1}(\theta \mid p,t)\, d\theta = \frac{p t^{p-1}}{\sum_{i=1}^{n} t_i^p}\left(\sum_{i=1}^{n}\delta_i\right) \tag{19}$$

- **Extension of Jeffreys prior information based on type-I censored data:** Extension of Jeffreys prior information is the Fisher information with the variable c where, c is a positive real number as:

$$g_2(\lambda) = \frac{k}{\lambda^{2c}} \tag{20}$$

The posterior probability density function of λ based on type-I censored data is:

$$\prod_{BE1}(\theta \mid p,t) = \frac{\left(\sum_{i=1}^{n} t_i^p\right)^{\sum_{i=1}^{n}\delta_i + 2c - 1}}{\theta^{\sum_{i=1}^{n}\delta_i + 2c}\left(\sum_{i=1}^{n}\delta_i + 2c - 2\right)!} \exp\left(-\frac{\sum_{i=1}^{n} t_i^p}{\theta}\right) \tag{21}$$

With this, the Bayesian for extension of Jeffreys prior estimates the scale parameter of Weibull distribution under squared error loss function are given as:

$$\hat{\theta}_{BE1} = \int_0^\infty A(\theta-\hat{\theta})^2 \prod_{BE1}(\theta \mid p,t)\, d\theta = \frac{\sum_{i=1}^{n} t_i^p}{\sum_{i=1}^{n}\delta_i + 2c - 2} \tag{22}$$

The estimator for survival function of Weibull distribution given shape by Bayesian using extension of Jeffreys obtained as:

$$\hat{S}_{BE1}(t) = \int_0^\infty \exp\left(-\frac{t^p}{\theta}\right)\prod_{BE1}(\theta \mid p,t)\,d\theta = \left(1+\frac{t^p}{\sum_{i=1}^{n} t_i^p}\right)^{1-2c-\sum_{i=1}^{n}\delta_i} \tag{23}$$

The estimator for hazard function of Weibull distribution by Bayesian using extension of Jeffreys obtained as:

$$\hat{h}_{BE1}(t) = \int_0^\infty \frac{p}{\theta} t^{p-1}\prod_{BE1}(\theta \mid p,t)\,d\theta = \frac{pt^{p-1}}{\sum_{i=1}^{n} t_i^p}\left(\sum_{i=1}^{n}\delta_i + 2c - 1\right) \tag{24}$$

Bayesian estimation of Weibull based on type-II censored data: The posterior probability density function of λ based on type-II censored data:

$$\prod_{BJ1}(\lambda \mid \alpha, x) = \frac{\frac{1}{\lambda}\left[\prod_{i=1}^{r}\frac{\alpha}{\lambda}x^{\alpha-1}\exp\left(-\frac{x_i^\alpha}{\lambda}\right)\right]\left[\exp\left(-\frac{x_r^\alpha}{\lambda}\right)\right]^{n-r}}{\int_0^\infty \frac{1}{\lambda}\left[\prod_{i=1}^{r}\frac{\alpha}{\lambda}x^{\alpha-1}\exp\left(-\frac{x_i^\alpha}{\lambda}\right)\right]\left[\exp\left(-\frac{x_r^\alpha}{\lambda}\right)\right]^{n-r} d\lambda} \tag{25}$$

With this, the Bayesian estimate for the scale parameter of Weibull distribution under squared error loss function is given as:

$$\hat{\lambda}_{BJ1} = \int_0^\infty A(\lambda-\hat{\lambda})^2\prod_{BJ1}(\lambda \mid \alpha, x)\,d\lambda = \frac{\sum_{i=1}^{r} x_i^\alpha + (n-r)x_r^\alpha}{r} \tag{26}$$

The estimate of the survival function of Weibull distribution given shape by Bayesian using Jeffreys prior is:

$$\hat{S}_{BJ1}(t) = \int_0^\infty \exp\left(-\frac{t^\alpha}{\lambda}\right)\prod_{BJ1}(\lambda \mid \alpha, x)\,d\lambda = \left(1+\frac{t^\alpha}{\sum_{i=1}^{r} x_i^\alpha + (n-r)x_r^\alpha}\right)^{-r} \tag{27}$$

The estimate of the hazard function of Weibull distribution based on type-II censored data by Bayesian using Jeffreys prior is obtained by:

$$\hat{h}_{BJ1}(t) = \int_0^\infty \frac{\alpha}{\lambda} t^{\alpha-1} \prod_{BJ1}(\lambda \mid \alpha, t)\, d\lambda = \frac{\alpha t^{\alpha-1}(r+1)}{\sum_{i=1}^{r} x_i^\alpha + (n-r)x_r^\alpha} \tag{28}$$

The posterior probability density function of λ based on type-II censored data:

$$\prod_{BE1}(\lambda \mid \alpha, t) = \frac{\frac{1}{\lambda^{2c}}[\prod_{r}^{i=1} \frac{\alpha}{\lambda} t^{\alpha-1} \exp(-\frac{t^\alpha}{\lambda})][\exp(-\frac{t^\alpha}{\lambda})]^{n-r}}{\int_0^\infty \frac{1}{\lambda^{2c}}[\prod_{r}^{i=1} \frac{\alpha}{\lambda} t^{\alpha-1} \exp(-\frac{t^\alpha}{\lambda})][\exp(-\frac{t^\alpha}{\lambda})]^{n-r} d\lambda} \tag{29}$$

With this, Bayesian with extension of Jeffreys prior estimate of the scale parameter of Weibull distribution based on type-II censored data are given as:

$$\hat{\lambda}_{BE1} = \int_0^\infty A(\lambda - \hat{\lambda})^2 \prod_{BE1}(\lambda \mid \alpha, t)\, d\lambda = \frac{\sum_{i=1}^{r} x_i^\alpha + (n-r)x_r^\alpha}{r + 2c - 1} \tag{30}$$

The estimate of the survival function of Weibull distribution given shape by Bayesian using extension of Jeffreys prior is obtained as:

$$\hat{S}_{BE1}(t) = \int_0^\infty \exp\left(-\frac{t^\alpha}{\lambda}\right) \prod_{BE1}(\lambda \mid \alpha, t)\, d\lambda = \left(1 + \frac{t^\alpha}{\sum_{i=1}^{r} x_i^\alpha + (n-r)x_r^\alpha}\right)^{1-r-2c} \tag{31}$$

The estimate of the hazard function of Weibull distribution by Bayesian using extension of Jeffreys prior based on type-II censored data obtained as:

$$\hat{h}_{BE1}(t) = \int_0^\infty \frac{\alpha}{\lambda} t^{\alpha-1} \prod_{BE1}(\lambda \mid \alpha, t)\, d\lambda = \frac{\alpha t^{\alpha-1}(r+2c)}{\sum_{i=1}^{r} x_i^\alpha + (n-r)x_r^\alpha} \tag{32}$$

Simulation study: To assess the performance of the maximum likelihood and Bayesian estimation based on type-I and II censored data to estimate the scale parameter follow by estimate survival and hazard functions. The Mean Squared Errors (MSE) was calculated using 10,000 replications for sample size n = 25, 50 and 100 of Weibull distribution with type-I and II censored data for different value of parameters were the scale parameter λ = 0.8 and 1.2, the shape parameter α = 0.5 and 1.5 and the two values of Jeffreys extension were c = 0.4 and 1.4, the considered values of parameters and extension of Jeffreys are meant for illustration only and other values can be taken for generating the samples from Weibull distribution.

RESULTS AND DISCUSSION

Four values of the estimators which are MLE, Bayesian using Jeffreys prior (BJ), extension of Jeffreys prior with c = 0.4 (BE(c = 0.4)) and extension of Jeffreys prior with c = 1.4 (BE(c = 1.4)) are shown in each column for each size.

Table 1 contains the estimate of the scale parameter of Weibull distribution based type-I censored data with maximum likelihood, Bayesian using Jeffreys prior and extension of Jeffreys prior information.

From Table 2 the results show that when c = 0.4, the maximum likelihood is better compared to the others with respect to the MSE of scale parameter. On the other hand, when c = 1.4, extension of Jeffrey is better compared to the others (Ahmed *et al.*, 2011). This implies that, as the value of extension of Jeffreys prior is kept below one, it exerts very minimal influence on the posterior distribution but as it increases to at least above one, the influence becomes significant on the posterior distribution from which Bayesian inference is drawn to give a very small mean squared error as compared to maximum likelihood and that of Jeffreys prior.

Table 3 shows the comparison of the survival function with the maximum likelihood is noticed to be better compared to the others when c = 0.4. On the other hand, when c = 1.4, extension of Jeffrey is better compared to the others following Ahmed *et al.* (2011).

Table 1: Estimate λ based type-I censored data

		$\lambda = 0.8$		$\lambda = 1.2$	
Size	Estimators	$\alpha = 0.5$	$\alpha = 1.5$	$\alpha = 0.5$	$\alpha = 1.5$
25	MLE	0.9022	0.7603	1.1817	1.3304
	BJ	0.9466	0.7563	1.1634	1.3972
	BE (c = 0.4)	0.9560	0.7590	1.1750	1.4114
	BE (c = 1.4)	0.8696	0.7838	1.1986	1.2814
50	MLE	0.9042	0.7655	1.1537	1.3230
	BJ	0.9284	0.7473	1.1332	1.3628
	BE (c = 0.4)	0.9334	0.7518	1.1393	1.3710
	BE (c = 1.4)	0.8858	0.7790	1.1812	1.2928
100	MLE	0.8990	0.7475	1.1269	1.3174
	BJ	0.9108	0.7280	1.1112	1.3366
	BE (c = 0.4)	0.9132	0.7301	1.1141	1.3405
	BE (c = 1.4)	0.8898	0.7594	1.1457	1.3024

Table 2: MSE for the scale parameter of Weibull based type-I censored data

		$\lambda = 0.8$		$\lambda = 1.2$	
Size	Estimators	$\alpha = 0.5$	$\alpha = 1.5$	$\alpha = 0.5$	$\alpha = 1.5$
25	MLE	0.0505	0.0331	0.0755	0.0916
	BJ	0.0668	0.0338	0.0766	0.1207
	BE (c = 0.4)	0.0709	0.0331	0.0778	0.1281
	BE (c = 1.4)	0.0412	0.0321	0.0748	0.0762
50	MLE	0.0292	0.0188	0.0381	0.0541
	BJ	0.0352	0.0192	0.0398	0.0648
	BE (c = 0.4)	0.0365	0.0198	0.0399	0.0672
	BE (c = 1.4)	0.0253	0.0165	0.0365	0.0474
100	MLE	0.0190	0.0121	0.0250	0.0344
	BJ	0.0214	0.0139	0.0260	0.0388
	BE (c = 0.4)	0.0219	0.0147	0.0266	0.0398
	BE (c = 1.4)	0.0172	0.0110	0.0239	0.0314

Table 3: MSE for the survival function of Weibull based type- I censored data

		λ = 0.8		λ = 1.2	
Size	Estimators	α = 0.5	α = 1.5	α = 0.5	α = 1.5
25	MLE	0.00280	0.00516	0.00199	0.00252
	BJ	0.00344	0.00553	0.00201	0.00271
	BE (c = 0.4)	0.00379	0.00569	0.00218	0.00282
	BE (c = 1.4)	0.00138	0.00492	0.00167	0.00241
50	MLE	0.00231	0.00362	0.00132	0.00123
	BJ	0.00262	0.00370	0.00142	0.00130
	BE (c = 0.4)	0.00278	0.00388	0.00144	0.00134
	BE (c = 1.4)	0.00148	0.00345	0.00116	0.00113
100	MLE	0.00199	0.00297	0.00118	0.00070
	BJ	0.00215	0.00298	0.00127	0.00074
	BE (c = 0.4)	0.00222	0.00304	0.00131	0.00075
	BE (c = 1.4)	0.00156	0.00252	0.00094	0.00062

Table 4: MSE for the hazard function of Weibull based type-I censored data

		λ = 0.8		λ = 1.2	
Size	Estimators	α = 0.5	α = 1.5	α = 0.5	α = 1.5
25	MLE	1.4149	0.1874	0.9731	0.0447
	BJ	1.4149	0.1874	0.9731	0.0447
	BE (c = 0.4)	1.4534	0.1957	1.0642	0.0448
	BE (c = 1.4)	0.9982	0.1203	0.5489	0.0435
50	MLE	1.2352	0.1270	0.6025	0.0230
	BJ	1.2352	0.1270	0.6025	0.0230
	BE (c = 0.4)	1.2496	0.1319	0.6597	0.0234
	BE (c = 1.4)	0.7312	0.1195	0.3962	0.0224
100	MLE	1.0045	0.1014	0.5352	0.0121
	BJ	1.0045	0.1014	0.5352	0.0121
	BE (c = 0.4)	1.1126	0.1291	0.5090	0.0123
	BE (c = 1.4)	0.4966	0.0949	0.1077	0.0112

As shown in Table 4 the hazard function estimates based on type-I censoring data was compared. The results show that when c = 0.4, the maximum likelihood is better compared to the others. Nevertheless, when c = 1.4, extension of Jeffrey has the smaller mean squared error as compared to others.

Table 5 estimates the scale parameter of Weibull distribution based type-II censored data with Maximum likelihood, Bayesian using Jeffreys prior and extension of Jeffreys prior.

Table 6 observed that, the mean squared errors of the Bayes estimator based type-II censored data under the extension of Jeffreys prior (c = 0.4) is the best estimator of the scale parameter for all the cases.

The survival function as indicated in Table 7 show that, the Bayesian using extension of Jeffreys based on type-II censored data is the best for all cases.

Table 8 show that, extension of Jeffreys prior based on type-II censored data is the best estimation.

Table 5: Estimate λ based type-II censored data

		λ = 0.8		λ = 1.2	
Size	Estimators	α = 0.5	α = 1.5	α = 0.5	α = 1.5
25	MLE	0.88213	0.73823	1.08343	1.33084
	BJ	0.89127	0.74118	1.08991	1.33964
	BE (c = 0.4)	0.90332	0.74292	1.09807	1.34882
	BE (c = 1.4)	0.79578	0.73391	1.07735	1.31825
50	MLE	0.75411	0.74141	1.10153	1.29384
	BJ	0.74426	0.74771	1.11723	1.30367
	BE (c = 0.4)	0.74926	0.75594	1.12765	1.31118
	BE (c = 1.4)	0.77213	0.73671	1.08994	1.27192
100	MLE	0.83115	0.75845	1.12928	1.28313
	BJ	0.84306	0.75952	1.13009	1.29771
	BE (c = 0.4)	0.84598	0.76078	1.14285	1.30743
	BE (c = 1.4)	0.82763	0.75810	1.11813	1.24576

Table 6: MSE for the scale parameter of Weibull based type-II censored data

		λ = 0.8		λ = 1.2	
Size	Estimators	α = 0.5	α = 1.5	α = 0.5	α = 1.5
25	MLE	0.00534	0.01130	0.01070	0.00692
	BJ	0.00546	0.01011	0.00976	0.00699
	BE (c = 0.4)	0.00571	0.00947	0.00950	0.00711
	BE (c = 1.4)	0.00491	0.01166	0.11013	0.00624
50	MLE	0.00510	0.00985	0.00932	0.00626
	BJ	0.00511	0.00975	0.00924	0.00632
	BE (c = 0.4)	0.00537	0.00963	0.00911	0.00676
	BE (c = 1.4)	0.00471	0.00998	0.00956	0.00604
100	MLE	0.00473	0.00831	0.00878	0.00611
	BJ	0.00485	0.00812	0.00853	0.00620
	BE (c = 0.4)	0.00507	0.00794	0.00886	0.00644
	BE (c = 1.4)	0.00456	0.00845	0.00919	0.00559

Table 7: MSE for the survival function of Weibull based type-II censored data

		λ = 0.8		λ = 1.2	
Size	Estimators	α = 0.5	α = 1.5	α = 0.5	α = 1.5
25	MLE	0.00258	0.00162	0.00302	0.00134
	BJ	0.00283	0.00130	0.00290	0.00147
	BE (c = 0.4)	0.00328	0.00120	0.00280	0.00149
	BE (c = 1.4)	0.00218	0.00174	0.00319	0.00120
50	MLE	0.00240	0.00155	0.00297	0.00116
	BJ	0.00260	0.00145	0.00273	0.00124
	BE (c = 0.4)	0.00300	0.00139	0.00261	0.00126
	BE (c = 1.4)	0.00153	0.00161	0.00301	0.00092
100	MLE	0.00160	0.00145	0.00255	0.00105
	BJ	0.00180	0.00136	0.00244	0.00117
	BE (c = 0.4)	0.00206	0.00129	0.00239	0.00122
	BE (c = 1.4)	0.00133	0.00146	0.00291	0.00086

Table 8: MSE for the hazard function of Weibull based type-II censored data

		$\lambda = 0.8$		$\lambda = 1.2$	
Size	Estimators	$\alpha = 0.5$	$\alpha = 1.5$	$\alpha = 0.5$	$\alpha = 1.5$
25	MLE	0.81403	0.15011	0.73425	0.03665
	BJ	0.83135	0.14128	0.72557	0.03759
	BE (c = 0.4)	0.84719	0.13684	0.71089	0.03885
	BE (c = 1.4)	0.79441	0.15109	0.74352	0.03335
50	MLE	0.80269	0.13128	0.71767	0.03495
	BJ	0.81552	0.12273	0.70387	0.03558
	BE (c = 0.4)	0.82368	0.12017	0.69133	0.03644
	BE (c = 1.4)	0.78603	0.14300	0.72536	0.02933
100	MLE	0.73024	0.09054	0.66949	0.02669
	BJ	0.73930	0.08814	0.65225	0.02729
	BE (c = 0.4)	0.74191	0.08587	0.63293	0.02911
	BE (c = 1.4)	0.71778	0.09171	0.67780	0.02436

CONCLUSION

In this study we have considered the Bayesian using extension of Jeffreys prior based on type-I and II censored data. The result show that, the Bayesian using extension of Jeffreys prior based on type-I is better estimator for all estimated of the scale parameter and the survival and hazard functions when the value of extension of Jeffreys is 1.4. On the other hand, the maximum likelihood method is better than others when the value of extension of Jeffreys is 0.4. The Bayesian using extension of Jeffreys prior based on type-II censored data is the best estimation compared to the other estimation.

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