Trends in
Applied Sciences Research

ISSN 1819-3579

Academic Journals Inc.

www.academicjournals.com

Trends in Applied Sciences Research 9 (1): 54-60, 2014
ISSN 1819-3579 / DOI: 10.3923/tasr.2014.54.60

Measurement Efficiency by Malmquist Method in Data Envelopment Analysis

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ABSTRACT

The Malmquist index is one of the most significant indices for the economists in evaluating efficiency of growth of all units. Up to now, lots of models have been introduced to develop the productivity index based on DEA (Data Envelopment Analysis). All these models have their own advantages and disadvantages. One of the most famous model is the Malmquist productivity index. One of the most important objection of this model is the mathematical complexity of it. For example, four different models should be solved to evaluate productivity progress of one Decision Making Unit (DMU) in two different years. So, if the DMUs or the evaluating years are increased, the mathematical complexity would increase, as well. In this study, it is tried to introduce a model with less complexity which can also make possible to compute productivity and technology changes in a simpler way. Therefore, a more acceptable analysis would come up, too.

Key words: Data envelopment analysis, productivity, malmquist, decision making unit

INTRODUCTION

In today's world, improving productivity amount is a national priority in every country. The reason is that, continuing the economic living, the economic progress and improving people's lives in e ach society depend on productivity growth. Malmquist (1953) talked about the word productivity and used this term as a quantitative index which is utilized in Data Envelopment Analysis (DEA). Then, it was developed by Fare and Grosskopf (1992) and Fare *et al*. (1994). After introducing DEA models by Charnes *et al*. (1978), those models were developed further such as the study performed by the same authors (Charnes *et al*., 1981). Fare composed the efficiency measurement idea which was offered by Fare and productivity measurement of Kao to obtain Malmquist productivity index. Fare and Lovell (1978) indicated that the gap function is on the contrary with the technical efficiency measurement and this makes it possible to use DEA model to calculate Malmquist index. After that, lots of researches have been carried out on this issue. In this study, using DEA and Malmquist productivity index, it is tried to compare the DMUs in groups and to represent the results of their progress or decline in different years.

A REVIEW OF DEA BASED MALMQUIST INDEX

In recent years, MPI is considered as a standard approach to measure the efficiency in the light of time changes. In MPI, to determine progress or decline of a DMU, it is not enough to compare the existing situation with its previous situation. The boundary of the studying society is also significant in its progress or decline, as well. To clarify this dual the following factors are offered:

Boundary changes = Current efficiency (t+1 time period)

Previous efficiency (t time period)

Efficiency boundary changes = Technology changes

Malmquist productivity index = Technology changes×the efficiency changes

DMUp efficiency with the characteristics of K time period comparing the boundary of L time period is shown as $\theta^L(X^k_p, Y^k_p)$ and it is obtained by the following model:

$$\begin{aligned} \theta^L(X^k_p, Y^k_p) = \quad & \text{Min } \theta \\ \text{s.t} \quad & \\ & \sum_{j=1}^{n} \lambda_j x^l_{ij} \le \theta x^k_{ip} && i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y^l_{rj} \ge y^k_{rp} && r = 1,\dots,s \\ & \lambda_j \ge 0 && j = 1,\dots,n \end{aligned} \tag{1}$$

In this model DMUp is not added to PPS of L time period. If one wants to add DMU^k_p to the PPS of the time period in the left-hand side of the constraint column $\lambda_{n+1}X^k_p$, $\lambda_{n+1}Y^k_p$, should be added and the following model should be used:

$$\begin{aligned} \theta^t(X^t_p, Y^t_p) = \quad & \text{Min } \theta \\ \text{s.t} \quad & \\ & \sum_{j=1}^{n} \lambda_j x^l_{ij} + \lambda_{n+1} x^k_{ip} \le \theta x^k_{ip} && i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y^l_{rj} + \lambda_{n+1} y^k_{rp} \ge y^k_{rp} && r = 1,\dots,s \\ & \lambda_j \ge 0 && j = 1,\dots,n \end{aligned} \tag{2}$$

where, $\theta^t(X^t_p, Y^t_p)$ is the DMUp efficiency at t time period comparing the boundary of it, in other words it is DMUp efficiency at t time period. Malmquist Productivity Index (MPI) for DMUp which is shown as MPIp or MP, equals: $MP = TEC_p \times FS_p$ in which, TECp represents the efficiency changes and it is defined as follows:

$$TEC_p = \frac{\theta^{t+1}\left(X^{t+1}_p, Y^{t+1}_p\right)}{\theta^t\left(X^t_p, Y^t_p\right)} \tag{3}$$

$$Fs_p = \left[\frac{\theta^t(X_p^t, Y_p^t)}{\theta^{t+1}(X_p^t, Y_p^t)} \cdot \frac{\theta^t(X_p^{t+1}, Y_p^{t+1})}{\theta^{t+1}(X_p^{t+1}, Y_p^{t+1})} \right]^{\frac{1}{2}} \tag{4}$$

Finally, after replacing and simplifying, the following MP definition is obtained:

$$M_p = \left[\frac{\theta^t(X_p^{t+1}, Y_p^{t+1})}{\theta^t(X_p^t, Y_p^t)} \cdot \frac{\theta^{t+1}(X_p^{t+1}, Y_p^{t+1})}{\theta^{t+1}(X_p^t, Y_p^t)} \right]^{\frac{1}{2}} \tag{5}$$

It should be considered that $\theta^{t+1}(X^t_p, Y^t_p)$ is DMUp efficiency in t time period, comparing t+1 boundary, also, $\theta^t(X^{t+1}_p, Y^{t+1}_p)$ is DMUp efficiency at t+1 time period, comparing t boundary. Moreover, $\theta^{t+1}(X^{t+1}_p, Y^{t+1}_p)$ is DMUp efficiency at t+1 time period, comparing t+1 boundary. As it is clear, to achieve Malmquist efficiency boundary, one should solve four models. Furthermore, to describe the technology changes of progress or decline of each DMU the following definitions are offered.

Definition 1: A part of a boundary moves positively, if and only if, this part at t+1 time period (comparing corresponding points in t time period), expands and enlarges PPS. It means that, the boundary gets larger in the exterior side.

Definition 2: A part of the boundary moves negatively, if and only if, this part at t+1 time period (comparing corresponding points at t time period) makes PPS smaller and boundary moves to the interior side. The case in which one part moves positively and the other one moves negatively, is possible, too. In this case, if the product of them was more than 1 the movement was positive and vice versa. So, in the technology dual three different cases may occur:

- **FSP>1:** Boundary movement is positive, or progress takes place
- **FSP<1:** Boundary movement is negative, or decline takes place
- **FSP = 1:** Boundary does not change

The efficiency has 3 different cases, too:

- **TEC>1:** It means efficiency at t+1 time period was more, comparing t time period. So, DMUp at t+1 time period is closer to t+1boundary, comparing t time period to its own boundary
- **TECp<1:** It means the efficiency at time period t is bigger than the efficiency in time period t+1. So, DMUp at time period t is closer to t boundary, comparing t+1 time period to t+1 boundary
- **TECp = 1:** Two efficiencies are equal

Therefore, the Malmquist productivity index which is the product of TECp and FSp has three different cases:

- **MP>1:** Productivity increased
- **MP<1:** Productivity decreased
- **MP = 1:** Productivity remains unchanged

In the above dual, Malmquist productivity index was calculated by the use of CCR model in input case.

METHODS

As it was expressed in the previous sections, the calculation of Malmquist index by using DEA approach, based on the comparison of the productivity growth of some units, is fulfilled along two periods. Now, if these periods were increased, for example, to ten, each two respective periods should be calculated individually. Now, if there is only one DMU, it is not possible to compute it because there's no comparative approach of DEA. Also, if DMUs increase, considering the comparison of each two respective time periods, the calculating complexity increases. All the mentioned models for productivity index try to answer two questions:

- How much progress does a DMU make, comparing its previous years?
- How much progress does a DMU make, comparing the best ones in the society?

In this study, a model is offered that not only answers to the above questions, but also takes less calculating complexity.

Suppose there are n DMUs to be compared in the light of m inputs and s outputs at K different time periods (K = t). First, the following model is offered to answer how a DMU is at K time period, comparing itself:

$$
\begin{aligned}
\text{Min} \quad & \theta_o^l \\
\text{s.t} \quad & \\
& \sum_{t=1}^{k} \lambda_t x_{io}^t \le \theta_o^l x_{io}^l && i = 1,\ldots,m \\
& \sum_{t=1}^{k} \lambda_t y_{ro}^t \ge y_{ro}^l && r = 1,\ldots,s \\
& \lambda_t \ge 0 && j = 1,\ldots,n
\end{aligned}
\tag{6}
$$

This model compares DMUo to itself, at t different time periods. θ^l_o indicates DMUo efficiency at L time period. The obtained efficiency of DMUo in different time periods shows the operation of DMU in these periods. If in all the periods the efficiency equal one, this implies that DMU operated efficiently in time period K. However, no progress or decline occurred. Otherwise, if in some periods DMU efficiently equals 1, it indicates DMU has the most efficient operation, comparing all the periods. Moreover, the progress or decline of DMUs is evaluated comparing the previous or the following efficiencies. In the next section, the details of this dual is presented. The mentioned model make possible to analyzing the progress or decline of only one DMU in different years. So, it does not have the objection of other models. Now, to answer the second question (i.e., how much progress does a DMU make comparing the bests in the society?) the following model is offered:

$$\begin{aligned} &\text{Min} \quad \eta_o^l \\ &\text{s.t} \\ &\sum_{t=1}^{k}\sum_{j=1}^{n} \lambda_{jt} x_{ij}^t \le \eta_o^l x_{io}^l && i=1,\ldots,m \\ &\sum_{t=1}^{k}\sum_{j=1}^{n} \lambda_{jt} y_{rj}^t \ge y_{ro}^l && r=1,\ldots,s \\ &\lambda_{jt} \ge 0 && j=1,\ldots,n \quad ,t=1,\ldots,k \end{aligned} \tag{7}$$

In the above model, all DMUs are compared to each other. This results in making the efficiency boundary by the best DMUs at the best time periods. Therefore, the progress or decline of each DMU at each time period is compared to the best possible boundary.

In fact, the first model comes up with the efficiency changes and the second one gives us the technology. Therefore, the new productivity index is defined as:

$$F_o = \frac{\theta_o^{l+k} \times \eta_o^{l+k}}{\theta_o^l \times \eta_o^l}$$

- **Fo =1:** It implies productivity at L and L+K time periods remains unchanged
- **Fo>1:** It indicates productivity progress at L and L+K time periods
- **Fo<1:** It represents productivity decline at L and L+K time periods

Using the above equation not only two consecutive years but also different years is compared with less computational burden. To do this it is enough to substitute η æ θ of those years into the equation and get their whole matrix of comparisons.

RESULTS AND DISCUSSION

Here, it is intended to utilize the model introduced in the previous section in order to calculate efficiency growth of total factors of National Iranian Oil Company during 1977-11988. The data consists of two inputs and one output. Inputs are representative of Capital and Labor and Output is the value added over the years to the National Oil Company. The separation of different years is given in Table 1 and their DEA efficiency score using GAMS software is given in Table 2. Also results of Table 2 are shown in Fig. 1.

Table 1: Summary statistics for the input and output data of total factors of National Iranian Oil Company

Year	Output	Input 1	Input 2
77	1000.000	1000.000	1000.000
78	86.191	103.769	103.107
79	77.236	103.458	104.425
80	47.521	101.572	105.232
81	48.924	100.321	103.417
82	69.998	100.985	110.351
83	70.603	102.675	113.724
84	63.890	102.953	116.474
85	64.384	101.025	117.586
86	60.381	99.293	117.951
87	64.025	96.503	117.798
88	66.405	93.669	123.109

Table 2:DEA efficiency for the input and output data of total factors of National Iranian Oil Company

Year	Efficiency	Year	Efficiency
77	1.00	83	0.69
78	0.84	84	0.62
79	0.75	85	0.64
80	0.47	86	0.61
81	0.49	87	0.66
82	0.69	88	0.71

Table 3: Productivity comparison in National Iranian Oil Company throughout years 77-88 with each other

Year	77	78	79	80	81	82	83	84	85	86	87	88
77	1.00	1.190	1.333	2.127	2.040	1.449	1.449	1.612	1.562	1.639	1.515	1.408
78	0.84	1.000	1.120	1.787	1.714	1.217	1.217	1.354	1.312	1.377	1.272	1.183
79	0.75	0.892	1.000	1.595	1.530	1.086	1.086	1.209	1.171	1.229	1.136	1.056
80	0.47	0.559	0.626	1.000	0.959	0.681	0.681	0.681	0.758	0.734	0.712	0.661
81	0.49	0.583	0.653	1.042	1.000	0.710	0.710	0.790	0.765	0.803	0.742	0.690
82	0.69	0.821	0.920	1.486	1.408	1.000	1.000	1.112	1.078	1.131	1.045	0.971
83	0.69	0.821	0.920	1.486	1.408	1.000	1.000	1.112	1.078	1.131	1.045	0.971
84	0.62	0.738	0.826	1.319	1.265	0.898	0.898	1.000	0.968	1.016	0.939	0.873
85	0.64	0.761	0.853	1.361	1.306	0.927	0.927	1.032	1.000	1.049	0.969	0.901
86	0.61	0.726	0.813	1.297	1.244	0.884	0.884	0.983	0.953	1.000	0.924	0.859
87	0.66	0.785	0.880	1.404	1.346	0.956	0.956	1.064	1.031	1.081	1.000	0.929
88	0.71	0.845	0.946	1.510	1.448	1.028	1.028	1.145	1.109	1.163	1.075	1.000

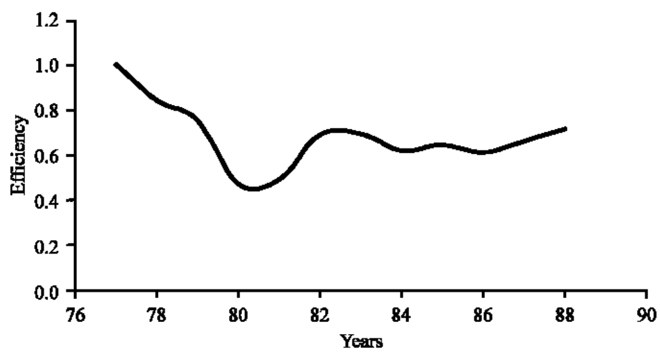

Fig. 1: Obtained efficiency based on Table 2

As can be seen, there is only one DMU in the example above. So, instead of θ, it is enough to insert number 1 in the numerator and denominator of the formula. The results of these comparisons can be seen in Table 3. Each line represents the efficiency ratio for the year compared to other years. Conversely, each column represents the efficiency ratio for other years compared to that year.

CONCLUSION

As it can be seen, the proposed method has the capability of non-consecutive years compared with less calculation's burden. Also there is a capability to compare a single decision maker in different years. In this method, the boundary of efficiency comes from the best DMUs in different years. Because all of the units in all the years under review form the possibility set. Thus, efficiency is calculated in different years using the best possible instances in the community. It seems that calculations using this method are more reasonable than Malmquist Method, because it may be a special year for all decision-making units in the population to have suffered from loss. In such a case, their comparison cannot provide us the correct measure of efficiency.

ACKNOWLEDGMENT

This study was performed within a research plan supported and funded by Islamic Azad University, Fars Science and Research Branch.

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