



Trends in
Medical Research

ISSN 1819-3587



Academic
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A Logarithmic Spiral Function to Plot a Cochleaogram

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Abstract: It is significant to create an appropriate cochleaogram (frequency-place map of a cochlea) to help hearing-impaired patients to achieve high level of speech and music recognition. A cochleaogram is also meaningful and useful in audiology, music therapy and music theory. Here, we originally propose a mathematical model of a logarithmic spiral function with a base 2, in a polar coordinates system, to plot a cochleaogram. The model is, $f_c(\theta) = f_{ms} \cdot 2^{\theta/\theta_0}$, where, $f_c(\theta)$ is a characteristic frequency of a cochlear basilar membrane, f_{ms} (2048 Hz) is the most sensitive frequency, θ is a spiral angle from the cochlear base to the cochlear apex (vice versa) around the origin of the system, θ_0 (90°) is a spiral period constant of octaves. The origin is assumed at the spiral center of the cochlea. We estimate the fitness between our modeling results and the published data. The averaged relative deviation is smaller than 33%. A cochlea works as an informative decoder that decodes the sound information from a frequency domain to a spiral spatial domain: $\theta(f_c) = \theta_0 \log_2(f_c/f_{ms})$. We also obtain perfect frequency ratios based on the semitone $2^{1/12}$ for all 12 tones logarithmically and spirally with our model.

Key words: Cochlea, implant, music, therapy, audiology, sound, acoustics, angle, octave, temperament

INTRODUCTION

Approximately 100,000 hearing-impaired individuals worldwide now have received cochlear implants and the contemporary devices clearly provide remarkable benefit to many implant recipients (Stakhovskaya *et al.*, 2007). It is significant to create an appropriate frequency-place map to help a given patient to achieve high level of speech and music recognition (Dorman *et al.*, 2007). Cochleaogram is just such a map to illustrate a relationship between characteristic frequencies and positions of a cochlear basilar membrane (Viberg and Canlon, 2004). The position parameters can be a (%) distance or an angle from the apex or the base (Stakhovskaya *et al.*, 2007; Dorman *et al.*, 2007).

According to Ohm's 2nd (acoustical) law, a pitch corresponding to a certain frequency could be heard only if the acoustic wave contained power at that frequency (Rossing, 2001). Békésy (1960) did many excellent experiments in hearing and systematically studied cochleaograms of distance positions with various species, such as a man, a guinea pig, a chicken, a rat, a mouse, a cow, an elephant. Scientists also found that a human cochlea has about 2 and 3 quarter turns (990°) totally (Kapit *et al.*, 1987).

Meanwhile, musician found when a frequency doubles, an octave results and human can hear about 11 octaves totally with a frequency range between 20 Hz and 20,000 Hz (Kapit *et al.*, 1987) or 38,000 Hz (Gorow, 2006).

Greenwood (1961, 1990) initially proposed a mathematical model with a base 10 logarithmic function of distance positions to plot a cochleaogram, assuming stimulation at the level of the hair cells. The model is very meaningful and very useful. However, the model does not have any spiral property. Hair cells are usually absent in implant patients after conventional implant surgery. The remaining

neural targets, dendrites and cell bodies in the spiral ganglion, do not have the same relationship as hair cells (Kawano *et al.*, 1996; Sridhar *et al.*, 2006). Moreover, the base 10 logarithmic function is not simple and elegant, compared with the base 2, to describe when a frequency doubles, an octave results. Therefore, the model is not likely to be appropriate for cochlear implant patients (Dorman *et al.*, 2007), it does not have any clear connection with music octaves and pitches and it does not fits experimental data well for the whole cochlear basilar membrane.

Early in the 20th century, a scientific pitch, with all C's being powers of 2 (128, 256, 512 and so on) appeared to describe octaves (Rossing, 2001). In some previous studies, scientists proposed 12 equal temperament using an exponential function with a base 2. The 12 tones are geometrically averaged (Rossing, 2001) or an octave is logarithmically divided to an equal interval (Gelfand, 2004): $2^{1/12} = 1.0595$. But, the relationships among a cochleaogram, a mathematical formula, octaves, 12 equal temperaments and perfect frequency ratios, have not been indicated.

In our previous study, we applied Ohm's 1st (electrical) law in a nerval signal encoding and a recognition processing of brain neurons about Alzheimer's disease (Cheng and Zou, 2007 a) and Coulomb's law in the informative responding during synthesis of DNA (Cheng and Zou, 2006) and proteins (Cheng and Zou, 2007b).

In this investigation, we originally propose a mathematical model of a logarithmic spiral function with a base 2, in a polar coordinates system, to plot a cochleaogram that covers the whole human auditory range, based on Ohm's 2nd (acoustical) law and published data. Our model mathematically predicates how a cochlea decodes sound information from a frequency domain to a spiral spatial domain. We also associate relationships among our model, cochleaogram, octaves and 12 tone equal temperaments. We believe our model is more meaningful and more useful than the previous to hearing-impaired patients and audiology, as well as music therapy and music theory. To our knowledge, there is not any model in the same way as that is elucidated in the present study.

MATERIALS AND METHODS

Based on Ohm's 2nd (acoustical) law and published data, we originally propose our mathematical model in a polar coordinates system to plot the cochleaogram:

$$f_c(\theta) = f_{ms} 2^{\theta/\theta_0} \tag{1}$$

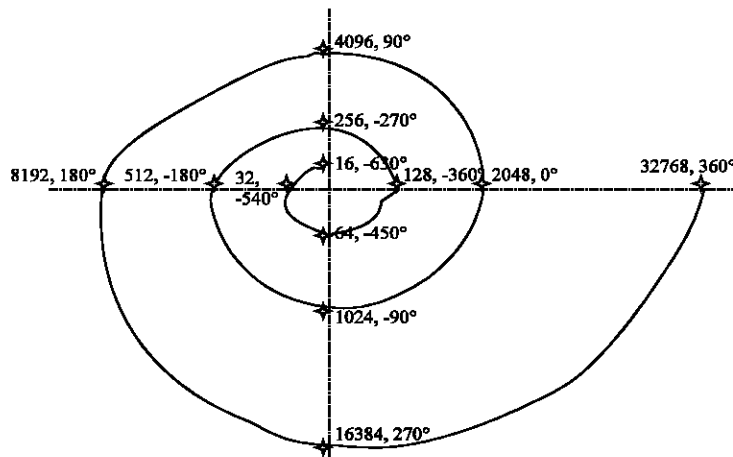


Fig. 1: A diagram of pairs of frequency (Hz) and angle (degree) of human cochlea in a polar coordinate system. The draw is not in scale

where, $f_c(\theta)$ is a characteristic frequency of the cochlear basilar membrane, f_{ms} is the most sensitive frequency, θ is a spiral angle from the cochlear base to the cochlear apex (vice versa) around the origin of the system, θ_0 is a spiral period constant of octaves. The origin is assumed at the spiral center of the cochlea. We, respectively chose f_{ms} to be 2048 Hz when $\theta = 0^\circ$ because the most sensitive frequency is between 1000 Hz and 4000 Hz (Kapit *et al.*, 1987) and θ_0 to be 90° because the human cochlea has total 990° and perceives 11 octaves, i.e., $990^\circ/11 = 90^\circ$. Figure 1 shows our model Eq. 1 and human cochlea. The bold data, i.e., $64 \text{ Hz} < \text{frequency} < 8192 \text{ Hz}$ and $-450^\circ < \theta < 180^\circ$, are the mostly musically effective.

RESULTS AND DISCUSSION

Figure 2 is a human cochleaogram. We illustrate the fitness between our modeling results and the published experimental data (Békésy, 1960; Kapit *et al.*, 1987; Gorow, 2006) in the Fig. 2. The averaged relative deviation is smaller than 33%.

In our mathematical model Eq. 1 in a polar coordinates system, θ presents the spiral structure of the human cochlea (including the basilar membrane and the sensory fibers); the base 2 indicates a frequency doubles when a pitch is highly octave repeated. Table 1 shows our modeling results of angles, frequencies, pitches and octaves. CI, I = -2, -1, 0, 1, ... 10, are music pitches, DI, EI, FI, GI, AI and BI are not shown in the figure for simplicity. The frequencies between 16 Hz and 32768 Hz are human perceivable. The frequencies between 64 Hz and 8192 Hz are the mostly musically effective. The deviation of the frequencies for each pitch between our modeling results and musician data (Gorow, 2006) are smaller than 3%.

A cochlea works as an informative decoder that decodes the sound information from a frequency domain to a spiral spatial domain. The decoding process follows the same Eq. 1 but in another form:

$$\theta(f_c) = \theta_0 \log_2(f_c/f_{ms}) \quad (2)$$

Then, the decoded codes, i.e., positions of the spiral angle θ are mapped to the auditory cortex through nerve system.

Using our model Eq. 1, we also study 12 equal temperaments. We divide 90° angle of an octave (e.g., 8th octave) into 12 equal angular interval, i.e., $90^\circ/12 = 7.5^\circ$ and increase a pitch when outward rotating an angle of 7.5° along a cochlear basilar membrane (Fig. 1). Then, we logarithmically

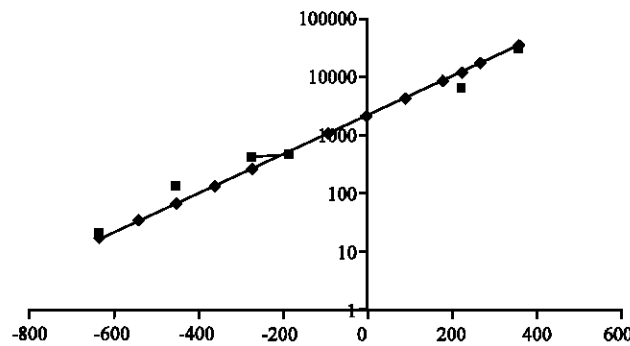


Fig. 2: A human cochleaogram and an illustration of fitness between our modeling results (◆) and the published data (■). The vertical and horizontal axes represent the frequencies (Hz) and the angles (degree), respectively

Table 1: Results of angles, frequencies, Pitches and octaves

Angle (°)	Frequency (Hz)	Pitch	Octave (Floor)	Octave (Ceiling)
-∞	0			
-810	4	C-3		
-720	8	C-2		
-630	16	C-1	1st	
-540	32	C0	2nd	1st
-450	64	C1	3rd	2nd
-360	128	C2	4th	3rd
-270	256	C3	5th	4th
-180	512	C4	6th	5th
-90	1024	C5	7th	6th
0	2048	C6	8th	7th
90	4096	C7	9th	8th
180	8192	C8	10th	9th
270	16384	C9	11th	10th
360	32768	C10	12th	11th
450	65536	C11	13th	12th
∞	∞			

Table 2: 12 equal temperaments of the 8th octave base on our modeling Eq. 1

Angle (°)	Frequency (Hz)	Pitch	Ratio Vs C6	Perfect Ratio Vs C6
0	2048	C6	1	0
7.5	$2048 \cdot 2^{1/12}$	C6#	$2^{1/12}$	1
15	$2048 \cdot 2^{2/12}$	D6	$2^{2/12}$	2
22.5	$2048 \cdot 2^{3/12}$	D6#	$2^{3/12}$	3
30	$2048 \cdot 2^{4/12}$	E6	$2^{4/12}$	4
37.5	$2048 \cdot 2^{5/12}$	F6	$2^{5/12}$	5
45	$2048 \cdot 2^{6/12}$	F6#	$2^{6/12}$	6
52.5	$2048 \cdot 2^{7/12}$	G6	$2^{7/12}$	7
60	$2048 \cdot 2^{8/12}$	G6#	$2^{8/12}$	8
67.5	$2048 \cdot 2^{9/12}$	A6	$2^{9/12}$	9
75	$2048 \cdot 2^{10/12}$	A6#	$2^{10/12}$	10
82.5	$2048 \cdot 2^{11/12}$	B6	$2^{11/12}$	11
90	$2048 \cdot 2$	C7	2	12

and spirally obtain corresponding frequencies, frequency ratios and perfect frequency ratios based on a semitone $2^{1/12}$, for all 12 tones (Table 2). All of the ratios are simple and elegant: 1, 2, ... and 12. The pitches are referenced from published musician data (Gelfand, 2004; Rossing, 2001).

Mathematically, our model Eq. 1 indicates how the characteristic frequency logarithmically and spirally increases while the spiral angle increases and how both octaves and temperaments are defined. It is very interesting that an ear could theoretically hear 0 Hz pitch if a cochlea turns to negative infinite based on the equation.

Clinically, we believe our model Eq. 2 will be more meaningful and more useful than the previous models to help hearing-impaired individuals to receive cochlear implants or to help a given patient to achieve high level of speech and music recognition.

Actually, the most sensitive frequency f_{ms} of a cochlea and the spiral period constant θ_0 of octaves must not be exactly the same for different patients or subjects, they are also slightly variant among different sections along a cochlea. These limitations of our model in this investigation are similar to that of the published models.

The principle of our model in this paper could be also helpful to plot a cochleaogram for other animals, such as guinea pigs, rats, mice, elephants. Of course, the most sensitive frequency f_{ms} of a cochlea and the spiral period constant θ_0 of octaves may be variant with different species. We can proximately obtain the parameters experimentally. e.g., for a rat with a cochlea of two and half turns (i.e., 900° in a total angle), if we choose two measurements of 20 and 80% of relative distances from the apex (Müller, 1991; Viberg and Canlon, 2004), to get the parameters, we have $f_{ms} = 1395$ Hz and $\theta_0 = 169^\circ$. The averaged relative deviation between our modeling data Eq. 1 and the experimental data is smaller than 30%.

REFERENCES

- Békésy, G.V., 1960. Experiments in Hearing. McGraw-Hill, New York.
- Cheng, K. and C.H. Zou, 2006. Informatics and physics models of recognitions of DNA replication and their biomedical applications. *Am. J. Applied Sci.*, 3 (10): 2059-2062.
- Cheng, K. and C.H. Zou, 2007a. Biomedicine and informatics model of Alzheimer's disease. *Am. J. Biochem. Biotechnol.*, 3 (3): 145-149.
- Cheng, K. and C.H. Zou, 2007b. Informatics models of recognitions of protein synthesis. *Asian J. Biochem.*, 2 (6): 432-436.
- Dorman, M.F. *et al.*, 2007. An electric frequency-to-place map for a cochlear implant patient with hearing in the nonimplanted Ear. *JARO.*, 8: 234-240.
- Gelfand, S.A., 2004. An Introduction to Psychological and Physiological Acoustics. 4th Edn. Marcel Dekker. New York.
- Gorow, R., 2006. Hearing and Writing Music. 2nd Edn. September Publishing, Studio City, CA.
- Greenwood, D.D., 1961. Critical bandwidth and the frequency coordinates of the basilar membrane. *J. Acoust. Soc. Am.*, 33: 1344-1356.
- Greenwood, D.D., 1990. A cochlear frequency-position function for several species-29 years later. *J. Acoust. Soc. Am.*, 33: 2592-2605.
- Kapit, W., R.I. Macey and E. Meisami, 1987. The Physiology Coloring Book. Murray, New York.
- Kawano, A., H.L. Seldon and G.M. Clark, 1996. Computer-aided three-dimensional reconstruction in human cochlear maps: Measurement of the lengths of organ of Corti, outer wall, inner wall and Rosenthal's canal. *Ann. Otol. Rhinol. Laryngol.*, 105 (9): 701-709.
- Müller, M., 1991. Frequency representation in the rat cochlea. *Hear Res.*, 51 (2): 247-254.
- Rossing, T.D., 2001. The Science of Sound. 3rd Edn. Addison-Wesley Publishing, USA.
- Sridhar, D., O. Stakhovskaya and P.A. Leake, 2006. A frequency-position function for the human cochlear spiral ganglion. *Audiol Neurootol.*, 1: 16-20.
- Stakhovskaya, O., D. Sridhar, B.H. Bonham and P.A. Lake, 2007. Frequency map for the human cochlear spiral ganglion implications for cochlear implants. *JARO.*, 8: 220-233.
- Viberg, A. and B. Canlon, 2004. The guide to plotting a cochleaogram. *Hearing Res.*, 197: 1-10.